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## Compensation of Processes with Time Delay

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**Abstract:** *Methods which allow comparisons in the use of PI and PID controller strategies for the control of first order lag plus time delay processes (FOLPD) are worthy of investigation because of the relative lack of work done in this area. In this paper, strategies for comparing performance and robustness for a PI or PID controlled FOLPD process are analysed and designed. The use of different PID controller structures for processes with time delay is also worth considering in detail. Various PID controller structures are compared by means of servo and regulator time responses, Bode plots and Nyquist plots.*

**Keywords:** PID, time delay, Integral of squared error, robustness and stability plots

### 1. Introduction

Processes with time delay occur frequently in chemical, biological, mechanical and electronic systems. Many high order systems with a time delay can be represented as an equivalent first order lag plus delay (FOLPD) model. There are many methods for approximating plant step responses with a FOLPD model such as that defined by Ziegler and Nichols [1]. The characteristics of the plant can then be expressed with three parameters, namely a gain  $K$ , time constant  $T$ , and deadtime  $h$ . The transfer function is given as:

$$Gp(s) = \frac{K \cdot e^{-h \cdot s}}{1 + T \cdot s}$$

The most common controller structure in process control applications is the PID or three term controller structure and its variations (P, PI or PD structure). Many strategies have been established in order to compare the performance and robustness of closed-loop control systems. Time domain based criteria for performance include percent overshoot, rise time and settling time of the transient response. More sophisticated performance criteria are the performance indices, like the ISE (Integral of the square of the error) index. The robustness of control systems can be described in terms of stability and sensitivity. Some of these strategies were analysed and designed for a ideal PI or PID controller in series with a FOLPD process. It is uncommon to implement the ideal PID or "textbook" algorithm in practice. The use of different PID controller structures to compensate processes with time delay is worth considering in detail. In this paper, the effects of various PID controller structures are compared by means of time domain based servo and regulator responses, Bode plots and Nyquist plots.

### 2. Performance and robustness issues of ideal PI and PID control of a FOLPD process

## 2.1 Analytical calculation of the Integral of squared error (ISE) performance index

The analytical calculation of the Integral of squared error for systems involving a time delay may be achieved using a method based on Parseval's theorem and contour integration [2]. This method is summarised as follows:

- The error  $E(s)$  of the system has to be calculated.
- It has to be proved that the system is asymptotically stable. A necessary, but not sufficient, condition for this is that the poles of  $E(s)$  lie in the open left half-plane.
- The calculation is easier in the frequency domain. Therefore, after using Parseval's

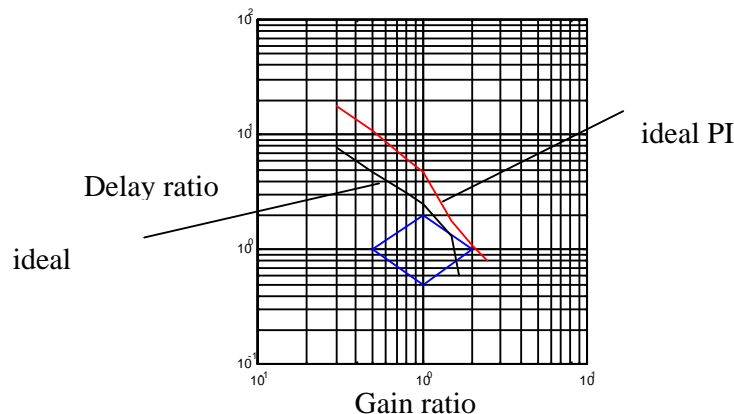
theorem, the ISE has the form: 
$$ISE = \frac{1}{2\pi j} \cdot \int_{-j\infty}^{j\infty} E(s) \cdot E(-s) ds$$

- Using the method of residues leads to one formula for the ISE. In [2] the formula is calculated assuming an ideal PI or PID controller in series with a FOLPD process (servo tuning); the conditions for asymptotical stability are also derived for the ideal PI and PID controller case (servo tuning).

These formulas and their development will be provided in detail on the poster.

## 2.2 Robustness and stability plots

Robustness is the ability of a controller to maintain closed-loop stability in the face of variations in process parameters [4]. A graphical representation of robustness are the robustness plots introduced by Gerry and Hansen [3]. This plot tells us how a loop will behave when the process time delay and gain change. The plot has a region of stability, and another of instability. Shinsky [4] converted the linear coordinate system used by Gerry and Hansen [3] into logarithmic coordinates to produce a parallelogram in the centre. The delay ratio equals the process time delay divided by the process time delay for which the controller was tuned. The gain ratio equals the process gain divided by the process gain for which the controller was tuned.



**Figure 1:** Robustness plot / ideal PI and PID / Ziegler-Nichols tuning /  $Gp(s) = \frac{1.75 \cdot e^{-12s}}{1 + 8s}$

The two lines in Figure 1 represent the limit of stability. To the right of them (higher gain ratios), the closed-loop system is unstable. To the left, the closed-loop system is stable. Where both ratios equal 1, the process gain and time delay are at the values for which they were tuned. In order to plot the stability lines the controller is first tuned for a base set of

process parameters. The process time delay is then changed in ratio to the nominal time delay until uniform oscillations are produced; this is the delay ratio at the limit of stability. Similarly, the process gain can be changed in ratio to the nominal gain until the limit of stability is again reached. Alternatively, both may be changed simultaneously. This means that a lot of simulations have to be done for each point of the stability line. The upper-right side of the parallelogram represents the locus of all products of delay and gain ratio equalling 2.0; similarly, the lower-left side represents all products equalling 0.5. The other two sides represent all products of the two ratios equalling either 2.0 or 0.5. Practically, a factor or divisor of two on the stability limit is desirable. If the stability limit for a control loop stays outside of this window, that loop is considered to be robust [4]. However, in [4] it is stated, that processes described by three or more parameters cannot be represented on this two-dimensional surface. In practice, however, the most changes occur in the process time delay and process gain. Therefore, it is appropriate to let the time constant of the FOLPD model be constant.

Alternatively, the authors have proposed and developed stability plots, representing the stable and unstable regions of the compensated process, corresponding to variations in the controller parameters. The stability plots are generated using the Bode criterion; no approximation for the time delay is required. The stability plots are two dimensional if a PI controller is used and are three dimensional if a PID controller is used. Full details will be provided in the poster.

### 3. Comparison of performance and robustness of various PI(D) structures for the compensation of FOLPD processes

The comparisons were made by means of servo and regulator step responses in the time domain (%overshoot, rise time, settling time), Bode plots (gain and phase margin) and Nyquist plots (maximum sensitivity). Notice that the maximum sensitivity was obtained by drawing a circle around the critical point -1, because the maximum sensitivity is the inverse of the shortest distance from the Nyquist curve to the critical point. This will be illustrated in the poster.

The controller structures, which were compared, are the ideal PI(D) controller, the PID controller with a filter on the derivative part, the industrial PID controller [5] and the 2 degree of freedom structure of the PI(D) controller with and without setpoint weighting [6]; these latter two controllers are indicated below.

$$\text{"industrial" PID algorithm: } U(s) = K_c \cdot \left(1 + \frac{1}{T_i \cdot s}\right) \cdot \left[ R(s) - \frac{1 + T_d \cdot s}{1 + \frac{T_d}{N} \cdot s} Y(s) \right]$$

$$\text{2 degree of freedom PID: } U(s) = K_c \left( \beta + \frac{1}{T_i \cdot s} \right) R(s) - K_c \left( 1 + \frac{1}{T_i \cdot s} + \frac{T_d \cdot s}{1 + \frac{T_d}{N} \cdot s} \right) Y(s)$$

Note:  $\beta$  = weighting factor

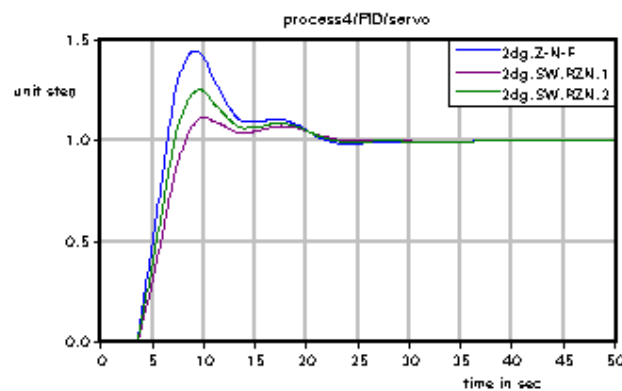
#### 3.1 Results

Sample simulation results, for the control of process  $G_p(s) = \frac{1.82}{1 + 7.68s} e^{-3.47s}$ , are summarised in the following table. Similar results for the control of six other processes will

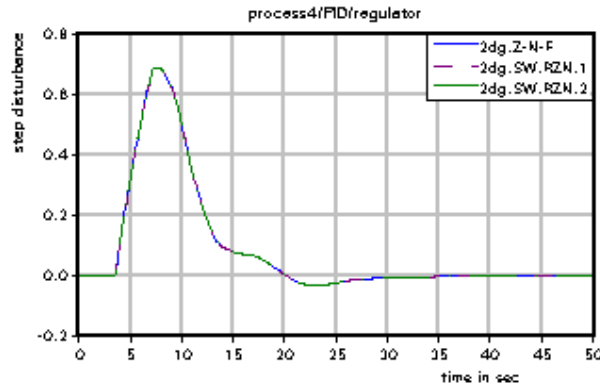
be provided in the poster. In the table, industrial refers to the industrial form of the PID controller, PID-filter to the PID controller with a filter on the derivative part, 2dg.-PI(D) to the 2 degree of freedom structure of the PI(D) controller without setpoint weighting and 2dg.SW.-PID to the 2 degree of freedom structure of the PID controller with setpoint weighting. For controller tuning, Z-N-P refers to the Ziegler-Nichols process reaction method [1], Z-N-F to the Ziegler-Nichols frequency response method [1], IAE to the minimum IAE tuning method [5], RZN to the refined Ziegler-Nichols tuning method [6] and  $\beta_1, \beta_2$  are different setpoint weighting factors [6];  $\beta_1$  corresponds to approximately a 10% overshoot in compensated system servo response, and  $\beta_2$  corresponds to approximately a 20% overshoot in response.

Controller	Tuning rule	%overshoot	rise time in sec	settling time in sec	gain margin in dB	phase margin in degree	sensitivity
ideal PI	Z-N-P	23	3.45	35.8	5.58	48.12	2.35
ideal PI	Z-N-F	23	3.6	30.96	6.15	48.47	2.35
2dg.-PI	Z-N-F	22	3.6	28.92	6.15	48.47	2.35
2dg.-PI	RZN	15	7.83	35	8.11	49.06	1.74
ideal PID	Z-N-P	57	1.25	26.1	3.41	51.04	3.13
ideal PID	Z-N-F	45	1.72	20	4.84	44.98	2.42
ideal PID	IAE	14	2.35	23.75	6.04	65.43	2.08
industrial	IAE	4	3.53	16.80	5.79	55.05	2.16
PID-filter	Z-N-P	62	0.94	35.5	2.77	50.48	3.62
PID-filter	Z-N-F	50	1.41	20.8	4.22	45.00	2.61
2dg.-PID	Z-N-F	45	2.20	21.6	4.22	45.00	2.61
2dg SW.-PID	RZN/ $\beta_1$	12	3.45	22.65	4.22	45.00	2.61
2dg.SW-PID	RZN/ $\beta_2$	25	2.98	22.65	4.22	45.00	2.61

**Table 1:** Simulation results / servo tuning



**Figure 2:** servo step response of the 2 degree of freedom structure (example) / different weighting factors



**Figure 3:** regulator step response of the 2 degree of freedom structure (example) / different weighting factors

The controller strategies have been implemented on a programmable logic controller (PLC), to compensate a laboratory test process. Details of these results will be provided in the poster.

#### 4. Conclusion

This paper has compared PID compensation strategies, using appropriate tuning rules, and has analysed methods to evaluate performance and robustness of these strategies, when applied to the control of FOLPD process models. An appropriate way to evaluate the performance is to analytically calculate the ISE or ITSE criterion. Robustness may be usefully evaluated using the robustness plots of Shinskey [4], by determining the PI stability plots or by graphical determination of the maximum sensitivity. The performance and robustness design requirements, as well as factors such as the ratio of time delay to time constant, will determine the best PID strategy to use. However, the two degree of freedom PID structure is generally recommended for the application, because of the flexibility the structure allows to optimise servo and regulator performance, which is borne out from the full panorama of test results.

#### 5. References

1. J.G. Ziegler, N.B. Nichols, "Optimum settings for automatic controllers", *Trans.A.S.M.E.*, 1942, 62, pp. 759-768.
2. J.E. Marshall, H. Goerecki, A. Korytowski, K. Walton, *Time-delay systems-stability and performance criteria with applications*, Publ. by Ellis Horwood Ltd., 1992.
3. J. Gerry, P. Hansen, "Choosing the right controller", *Chem. Eng.*, May 25, 1987, pp.65-68.
4. F.G. Shinskey, *Process control systems*, Mc Graw-Hill Book Co., 3<sup>rd</sup> Edition, 1988.
5. A. Kaya, T.J. Scheib, "Tuning of PID Controls of Different Structures", *Control Engineering*, July 1988, pp.62-65.
6. Hang, K.J. Astrom, W.K. Ho, "Refinements of the Ziegler-Nichols tuning formula", *IEE Proceedings*, Vol. 138, No.2, March 1991, pp. 111-118