Optimisation Methods in Supply Chain Applications: a Review

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OPTIMISATION METHODS IN SUPPLY CHAIN APPLICATIONS: A REVIEW

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OPTIMISATION METHODS IN SUPPLY CHAIN APPLICATIONS: A REVIEW

ABSTRACT

The competitiveness and dynamic nature of today’s marketplace is due to rapid advances in information technology, short product life cycles and the continuing trend in global outsourcing. Managing the resulting supply chain networks effectively is a complex and challenging task which is imputable to high level of uncertainty in supply-demand, conflict objectives, vagueness of information, numerous decision variables and constraints. With such level of complexity in the environment, supply chain optimisation has a potential to make a significant contribution to resolve the challenges. In this paper, a literature review – based on more than one hundred peer-reviewed articles – of state-of-the-art optimisation techniques in the context of supply chain management is presented. It also provides a classification of solution techniques. Linear programming, integer programming and mixed-integer programming have been used to solve many issues including; facility location, demand allocation and vehicle routing problems. The aforementioned traditional techniques have limited capabilities to handle the inherent interdependencies in supply chain networks. Such limitations of different optimisation techniques are discussed in detail. As a result, trends in current optimisation methodologies are based not only on improving a particular process performance but also on achieving a broader impact on supply chain efficiency. When properly applied, these methodologies can create precise and comprehensive models of great practical value for decision makers in managing supply chains. In such a vigorous global marketplace, supply chain optimisation is no longer an option; it is a requirement for survival to remain competitive.
1 INTRODUCTION

Experience and intuition was often the base of most critical decisions in enterprises. However, in today’s dynamic marketplace, these decisions are far from optimum and lead to a deteriorated performance. Proper selection of suppliers, distribution centres, facilities and equipments are few examples of challenging tasks that face managers at different levels in supply chain. These challenges emerges from the increasing complexity of supply chain networks which is imputable to high level of uncertainty in supply-demand, conflict objectives, vagueness of information, numerous decision variables and constraints.

Managers make decisions at different levels in supply chain. These decisions are needed to be supported by robust tools to enable managers to evaluate the impact of their decisions prior to actually making them in the real environment. System modelling (Aguilar-Savén, 2004) is used in such cases to model the real system (Figure 1). These models can be mathematical models or simulation models. In order to capture the system complexity, mathematical models are rarely used to model supply chain. The interaction between supply chain components (i.e., manufacturing plants, retailers, warehouses, and distribution centres) defines its complicated behaviour and is very difficult to build analytical expression that describes it precisely. Simulation is one of the most successful tools for analysing supply chain processes (Terzi and Cavalieri, 2004). Compared to analytical techniques, simulation provides the flexibility to accommodate arbitrary stochastic elements, and generally allows modelling of all complexities and dynamics of real-world supply chains without undue simplifying assumptions.

Model specifications represent the input parameters \( (x_i) \) (Figure 1. Mapping of a real system to a Model. These parameters are the decision variables that managers need to set their values such as number of facilities and location of distribution warehouse. \( F \) is the
model output – performance criterion (e.g., total cost or resource utilization) that shows the impact of changing variables.

**Figure 1. Mapping of a real system to a Model using Modelling Techniques**

Since simulation mimics reality, it permits the inclusion of uncertainty and variability into the model; this makes simulation suitable to supply chains which are characterized by high-level of variability. Furthermore, it considers supply chain constraints into the model (i.e., capacity constraints, manufacturing due dates, maximum holding time for backorders) (Figure 2).

As a complex dynamic system, the number of decision variables in supply chain network can be immense. Hence, simulating all possible combinatorial options is not possible. Optimisation therefore is essential to find a set of values for the decision parameters that leads to optimal performances. While optimisation tools allow the user to find the best possible solutions, simulation is used to evaluate individual solutions (Figure 3). Thus integrating simulation and optimisation in supply chain framework provides the decision makers with a comprehensive solution toolbox.
2 PROBLEM FORMULATION

The resemblance of supply chains to dynamic engineering systems is extremely helpful for building an integrated management framework. Most business problems can be described as:
optimise \[ f_i(x) \quad i = 1, \ldots, I, \]
subject to: \[ g_j(x) \leq 0 \quad j = 1, \ldots, J, \]
\[ h_k(x) = 0 \quad k = 1, \ldots, K. \]

Where \( f_i(x) \) is the objective function \( i \), \( x \) represents decision variables vector, and \( g_i(x), h_i(x) \) are the set of inequality and equality constraints. Finding the set of values of decision parameters \( x \) that optimise (minimise/maximise) the interested performance criterion \( f \) faces many challenges. Firstly, obtaining a mathematical description of \( f_i(x) \) is not attainable. This is due to the unclear relationships between the system components that define the behaviour (output) of the system. That is the reason why the use of traditional optimisation techniques – in particular those usually use analytical tools to get the optimum values of the variables – are not adequate for complex problems. Secondly, supply chains are usually characterised by multi-objectives that tend to be conflict (Min and Zhou, 2002). Maximum resource utilisation, minimum cost, and minimum lead time are an example of conflict objectives. These objectives have to be optimised concurrently to provide effective solutions. Moreover, optimisation methods have to consider the uncertainty embedded in supply chain to provide reliable solutions (Van der Vorst and Beulens, 2002). This makes traditional techniques recall the need for adjustments to deem different sources of uncertainty in the underlying system. Finally, due to the complexity of supply chain networks modelling, a large number of variables are encountered with large ranges. This leaves the system optimiser with very large number of parameter combinations that are infeasible to enumerate or simulate. These kind of NP-Hard problems (Pardalos, 2005) need more sophisticated optimisation algorithms that guide the search for best solutions in a reasonable timeframe.

The purpose of this paper is to comprehensively review the literature on supply chain optimisation. This literature review categorises optimisation methods in a unified
classification. The proposed classification summa ries optimisation approaches and its applications in supply chain.

3 APPROACHES AND APPLICATIONS

Objective function or performance measure mostly cannot be described using a mathematical model because of the inherited certainty level. Simulation models are consequently used to evaluate the different configurations of the system to be optimised. This type of optimisation is known as simulation optimisation in Operation Research (OR) literature (Tekin and Sabuncuoglu, 2004).

As a result, $f(x)$ is usually estimated through simulation model, e.g., the output of the simulation gives $\hat{f}(x)$. Methods used to optimise this stochastic objective functions are called direct search methods, because the uncertainty is treated directly by optimising stochastic functions (Beyer and Sendhoff, 2007). Figure 4 shows different techniques used for stochastic-function optimisation which is described in detail in the following subsections.

3.1 Gradient-based Methods

Differentiation in the gradient context is usually used to simplify the objective function in order to find an optimum solution. Gradient-based approach is subject to have a mathematical expression of the objective function. When such mathematical expression cannot be obtained, there is a need to use an estimation technique to start the solution procedure. The estimated gradients direction guides the search process to move from one potential solution to another in an iterative scheme in a process called stochastic approximation (SA) (Robbins and Monro, 1951). During the iteration phase, the step size is controlled by the gradient estimator which is embedded in optimisation algorithms.
Perturbation Analysis (PA), Finite Difference Estimation (FDE), and Likelihood Ratio Estimator (LRE) are three gradient estimators were successfully simulation optimisation.

Figure 4. Proposed Direct Search Methods Classification

3.1.1 Perturbation Analysis (PA)

PA has been applied to a large number of applications such as; queuing systems (Ho, 1984, Ho, 1985), inventory systems (Fu and Healy, 1997). Infinitesimal PA (IPA) is used
to determine asymptotically unbiased gradient estimates for computing the minimum average network delay in intree ATM networks (Brooks and Varaiya, 1997). Smoothed PA was used to optimise threshold values of repair times in a maintenance model (Heidergott, 1999).

Infinitesimal PA (IPA) is considered unbiased gradient estimation (Glasserman, 1991). The convergence rate has been studied in (L’Ecuyer and Perron, 1994) while variance reduction and efficient implementation of IPA was investigated in (Dai, 2000). IPA was applied to a stochastic fluid model (SFM) in order to capture the operation of threshold-based production control policies in manufacturing systems (Yu and Cassandras, 2004). PA estimates all gradients of the performance measure of interest by tracking the propagation of simulation output sensitivity through the system when a decision variable is perturbed slightly (Ho, 1984). To have these tracking capabilities in the simulation model, an understanding of the simulation model is required to allow system optimisers to integrate their algorithms into the model. This adds difficulty for applying PA to simulation optimisation (Azadivar, 1999). Simultaneous perturbation stochastic approximation (SPSA) overcomes this problem by treating the simulation model as a black box (Spall, 2005).

3.1.2 Finite Difference Estimation (FDE)

FDE is based on determining partial derivatives of the performance measure of the problem (Pegden and Gately, 1977). In order to estimate the gradient at each search point, at least \((n + 1)\) evaluations of the simulation model are necessary, where \(n\) is the number of decision variables. For a more reliable estimate, multiple observations for each derivative are required.

3.1.3 Likelihood Ratio Estimator (LRE)

LRE estimates the derivative of the performance measure by mathematically differentiating the underlying probability measure of the system (Glynn, 1990). LRE is applicable for large classes of reliability and queuing models (Nakayama and Shahabuddin, 1998). This
The applicability of LRE can be studied and verified using generalised semi-Markov processes (GSMP) which is a general mathematical framework for modelling many discrete-event systems (Glasserman and Yao, 1992). Combining PA and LRE to derive estimators in optimisation algorithms were tested in the case of using Monte Carlo Simulation and found effective (Fu and Hu, 1999).

3.2 Metamodel-Based Method

While gradient-based estimators are used to estimate the derivatives of the objective function, metamodel-based techniques use analytical approach to approximate the objective function itself. The analytical model is developed based on the relationship between the decision variables (i.e., input) and the simulation model output (Figure 5).

A metamodel simplifies the optimisation process in two ways;

- metamodel output is not stochastic (i.e., methods of optimisation can directly be applied), and
- computation cost in terms of time is significantly reduced

Figure 5. Metamodel interacts with the simulation model and the optimisation module
There are different types of metamodels such as polynomial regression models (i.e., linear regression), artificial neural networks (i.e., non-linear regression), support vector machines and kriging (i.e., interpolation).

3.2.1 Response surface Methodology (RSM)

RSM is based on procedures that allow regression models to be applied to simulation models responses that evaluated at several values of decision variables using design of experiments (DOE) before the resulting regression function is optimised. A comprehensive study on the use of statistical designs integrated with simulation models can be found in (Kleijnen, 1998). This study focuses on how RSM combines regression analysis, statistical designs and the steepest (descent/ascent) method to optimise the objective function of the simulated system. In manufacturing, RMS combines with simulation models is used as a systematic approach for design and analysis of manufacturing cells. It facilitates the design process and the allocation of resources by identifying the settings of cell design and operating factors (e.g., maintenance policy, quality policy, lot size, number of operators) that optimise the performance (Irizarry et al., 2001). A key advantage of RSM is its ability to optimise objective function with unknown variance along with high level of uncertainty (Nicolai et al., 2004). Moreover, RSM can be extended to allow multiple random system responses with multi constraints which so called Generalised RSM (Kleijnen, 2008). A further detailed description about RSM and its applications in real-time systems can be found in (Barton and Meckesheimer, 2006).

3.2.2 Kriging

Kriging: is an interpolation method that predicts unknown values of a random function. They are more flexible than polynomial models in fitting arbitrary smooth response functions, and seem to be less sensitive to small changes in the experiment design
(Meckesheimer et al., 2002). It was successfully applied for sensitivity analysis of stochastic simulation with a single output (Kleijnen and Van Beers, 2004) and offers opportunities for solving constrained optimisation problems (Biles et al., 2007). They provide a description of experiments in order to explore the potential of this methodology for constrained simulation optimisation. The experiments study an (s, S) inventory system with an objective of finding the optimal values of s and S (s denotes reorder point, S denotes maximum inventory level).

3.2.3 Artificial Neural Networks (ANNs)

Artificial Neural Networks (ANNs) proven to be an effective method to approximate arbitrary smooth functions and can be fitted using stochastic response values (Haykin, 2008). ANNs were developed to mimic neural processing, whose inputs and outputs are linked according to specific topologies. A special attention for the ANN training set has to be given to avoid over-fitting approximation that directly affects the predictive accuracy resulting out of ANN. Design of experiments can be combined with ANN to overcome over-fitting problem (Alam et al., 2004).

3.3 Statistical Methods

Gradient-based and metamodel-based optimisation techniques are used in case of continuous parameters. In discrete decision parameters, the problem is to select one of the predetermined system configurations (i.e., there is a finite solution space). The task of optimisation algorithms is to select one of these configurations that achieve the best performance of the system based on the selected criteria. Since the system performance is not deterministic and so as the output of a stochastic simulation model, further statistical analysis is required to compare alternatives. Different types of approaches were developed for such
optimisation problems, including Ranking & Selection (R&S), Multiple Comparison Procedures (MCP), and Ordinal optimisation (OO).

3.3.1 Ranking & Selection (R & S)

There are two main approaches with respect to ranking and selection. The first is the *indifference-zone* approach. It finds the values of decision variables that makes the value of performance measure differs from the optimal performance by at most a small amount (i.e., indifference zone). On the other hand, *Subset selection* (the second approach) is used to reduce the feasible solution region to a small subset (i.e., It aims to select a subset of at most $m \leq k$ where $k$ is the total number of possible alternatives and guarantees that this subset contains at least the best solution). Indifference-zone doesn’t require extensive computation efforts and can be applied to a single replication from each solution (Kim and Nelson, 2001, Kim and Nelson, 2006).

As the number of solutions (i.e., system configurations) increases, the sampling efforts increases which deteriorate the performance of indifference-zone. A general theory and procedures are used to reduce sampling efforts in indifference-zone approach when the number of alternatives is large (Nelson et al., 2001).

For systems with multiple performance measures, multi Attribute utility theory can be combined with the indifference-zone to find the best system configuration (Butler et al., 2001).

3.3.2 Multiple Comparison Procedures (MCP)

The idea of *Multiple Comparison Procedures (MCP)* is to run a number of replications and then evaluate make conclusions on a performance measure by constructing confidence intervals. In general, for all alternative pairs, the differences between the
estimates of the performance measures are computed and $(1 - \alpha)100\%$ confidence intervals are formed for each interval. The system corresponding to the confidence interval for which the differences with all other pairs are strictly negative is selected (Hochberg and Tamhane, 1987, Swisher et al., 2003).

3.3.3 Ordinal Optimisation (OO)

Sometimes it is difficult to precisely determine the best alternative from a set of predefined solutions in terms of absolute values. It is much easier to estimate approximate relative order. Ordinal Optimisation (OO) determines which solution is better rather focusing on the quantitative difference between the available solutions. In addition, instead of looking for the best alternative, OO selects a good enough solution (goal softening) (Ho et al., 2000). This crucial feature of OO makes it a robust optimisation choice when the number of alternatives is very large (Lin et al., 2004, He et al., 2007, Chen, 2004).

3.4 Random Search / Metaheuristics

Statistical methods were successfully used in the case of discrete decision parameters. However, it is computationally infeasible to evaluate every possible alternative or all parameter combinations when the solution space is very large. Determining which alternative(s) to be simulated and evaluated is crucial. Besides, most of the optimisation techniques mentioned in previous sections fail to find an optimum solution when the solution space is high-dimensional, discontinuous, or when the decision variables are qualitative. Metaheuristics are used in such cases to efficiently guide the search process towards potential solutions points (Doerner et al., 2007). They ultimately balance between exploration of solution space and exploitation of good solution(s) to overcome the conflict between local optimum solutions and the global ones. This is performed in an iterative process by initially
start with a solution (point-based) or set of solutions (set-based or population based), then in each iteration the search progresses to a new possibly better solution(s) in the neighbourhood of the current solution. Each metaheuristic method has its own mechanism to define the neighbourhood structure (Andradottir, 2006). Recently, metaheuristics has been widely used in solving supply chain problems (Table 1).

3.4.1 Genetic Algorithms (GAs)

GAs work on a population of solutions in such a way that poor solutions excluded, whereas the good solutions evolve to reach for the optimum (Mitchell, 1996, Goldberg, 1989). It generates initial population of solutions. These solutions are then evaluated through a simulation model which is followed by a selection process in which genetic operators are applied to produce a new offspring (or solution) which is inserted into the population (Figure 6). This process is repeated repeat until some stopping criterion is reached (Mitchell, 1996, Goldberg, 1989).

![Figure 6. Genetic Algorithm (GA) uses simulation model as a black box for fitness computation](image)
GAs are successfully used to numerous supply chain problems such as warehouse layout problem (Zhang et al., 2002, Min et al., 2005), order distribution (Chan et al., 2004), inventory management (Daniel and Rajendran, 2005, Noorul Haq and Kannan, 2006), Supplier selection (Ding et al., 2005), spare part allocation (Marseguerra et al., 2005), supply chain configuration (Truong and Azadivar, 2005), multi-constraint scheduling (Dawood and Sriprasert, 2006, Silva et al., 2008), bullwhip effect (O’donnell et al., 2006), and logistics (Yin and Khoo, 2007).

### 3.4.2 Simulated Annealing (SA)

SA starts with an initial solution, generally chosen randomly. A neighbour of this solution is then generated by a suitable mechanism. The performance of this solution is then calculated. If an improvement occurs, the generated neighbour replaces the current solution. If there is no improvement in the performance, the SA algorithm may accept this solution with some probability to avoid entrapment in a local optimum (Kirkpatric et al., 1983). SA is used to a broad spectrum of applications in industry such as buffer allocation in production lines (Spinellis and Papadopoulos, 2000, Cave et al., 2002), process scheduling (Spinellis and Papadopoulos, 2000, Cave et al., 2002). It was also used to determine the vehicle routes for transportation planning problems (Kim et al., 2002). SA proven to be efficient in scheduling problems that are nonlinear, non-convex with large numbers of variables (McCormick and Powell, 2004). Besides its ability to solve multi-objective optimisation problems taking into consideration the system constraints (Allaoui and Artiba, 2004).

### 3.4.3 Tabu Search (TS)

TS is a constrained search procedure, where each step consists of solving a secondary optimisation problem (Glover and Laguna, 1997). At each step, the search procedure removes a subset of the search space. This subset changes as the algorithm proceeds and is usually
defined by previously considered solutions which are called the reigning tabu conditions (Chelouah and Siarry, 2000). It is successfully used for optimisation problems such as scheduling problems (Grabowski and Wodecki, 2004, Geyik and Cedimoglu, 2004) and vehicle routing (Fu et al., 2004, Vogt et al., 2006).

### Table 1. Metaheuristics Applications in Supply Chain Problems

<table>
<thead>
<tr>
<th>Supply chain problem</th>
<th>Description</th>
<th>Objective Function(s)</th>
<th>References</th>
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| Logistics            | Rescheduling of a logistic system | - Minimise the number of orders that are not delivered and are already delayed  
- Maximise the number of orders delivered at the correct date | (Silva et al., 2008) |
| Logistics            | Vehicle routing considering multiple depots, multiple customers, and multiple products | - Minimise total travelling distance  
- Minimise total travelling time | (Lau et al., 2008) |
| Logistics            | Routing selection and operation sequence | - Minimise cost  
- Maximise on time delivery | (Yin and Khoo, 2007) |
| Job-Shop Scheduling  | Scheduling of various types of equipments simultaneously in container terminals | - Minimise time being taken to load or unload a given set of outbound containers | (Zeng and Yang, 2009) |
| Job-Shop Scheduling  | Machine scheduling in manufacturing system | - Minimise the maximum completion time  
- Minimise the maximum tardiness | (Pan et al., 2008) |
| Job-Shop Scheduling  | A multi-attribute combinatorial dispatching (MACD) problem in an flow shop with multiple processors (FSMP) environment | - Minimise the maximum  
- Tardiness | (Yang et al., 2007) |
| Production Planning  | Optimize the production planning and control policies for dedicated remanufacturing | - Maximise the total profit | (Li et al., 2008) |
| Allocation           | Human resource allocation | - Maximise the profit  
- Minimise the total cost | (Lin and Gen, 2008) |
| Allocation           | Spare parts allocation problem | - Minimise Cost  
- Minimise fill-rate | (Lee et al., 2008) |
| Allocation           | Dynamic facility location (Warehouse) | - Minimise operating costs of warehouses  
- Minimise transportation costs | (Ko et al., 2006) |
| Allocation           | Reliable construction schedules | - Minimise project duration and cost  
- Maximise resource and space utilization | (Dawood and Sriprasert, 2006) |
| Allocation           | Allocation of customers to warehouses in a balanced way | - Minimise the total costs, while balancing costs at each warehouse as equitably as possible | (Min et al., 2005) |
| Allocation           | Spare parts allocation and handling policy | - Maximise the net profit  
- Minimise the total spares volume | (Marseguerra et al., 2005) |
| Bullwhip             | Determine the optimal ordering policy for members of the SC | - Minimise the total cost | (Kumar et al., 2007, O’donnell et al., 2006) |
| Supplier Selection   | Supplier selection and multi-echelon distribution inventory in a built-to-order supply chain environment | - Minimisation of the total SC costs | (Noorul Haq and Kannan, 2006) |
Aforementioned metaheuristic techniques use simulation model as a black box to measure the impact of the selected solution on the system performance. As these techniques need extensive evaluation of objective function, it is very slow when used for simulation optimisation. Another difficulty associated with applying metaheuristics is the stochastic nature of its internal mechanism used for finding and selecting potential solutions. This adds another stochastic factor to the optimisation process. Moreover, the convergence rate of these techniques is considerably slow and cannot be mathematical proven.

**CONCLUSION**

Optimisation techniques have showed a great potential to solve supply chain management problems that causes an immense challenge to decision makers. These challenges are imputable to high level of uncertainty in supply-demand, conflict objectives, lack of needed information, numerous decision variables, and inevitable constraints. Traditional techniques (e.g., linear programming, integer programming and mixed-integer programming) have limited capabilities to handle the inherent interdependencies in current supply chain networks. This paper discussed various optimisation approaches and presented their applications in supply chain context. Optimisation methodologies have to focus not only on improving a particular process performance but also on achieving a broader impact on supply chain efficiency. Accordingly, metaheuristics algorithms were presented into supply chain applications because of its global optimisation capabilities in stochastic environments. On the other hand, statistical methods and metamodel-based methods can be incorporated with metaheuristics to provide more reliable solutions in a reasonable timeframe.

During last few decades, many optimisation techniques were found and made available to system analysts. A classification of the current optimisation techniques is provided in this
paper that takes into consideration the optimisation mechanism used to handle the problem, the decision parameters (continuous or discrete) and the search space (finite or infinite). The absence of a clear guideline that considers other factors (e.g., constraints handling, multi-objective and robust solutions) makes the decision to select an optimisation technique considerably hard.

References


