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Modelling and Control of a Suspension System for Vehicle Applications

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Abstract: This paper discusses the modelling of passive and active suspension systems in a car, and the subsequent design of appropriate feedback controllers for the active suspension system. The models will be investigated using a quarter car model and a full car model approach.

1. Introduction
The main objective of suspension systems is to reduce motions of the sprung mass (vehicle body) to road disturbances. Conventional vehicle suspension systems achieve this through passive means using springs and dampers. When designing vehicle suspensions, the dual objective is to minimise the vertical forces transmitted to the passenger, and to maximise the tyre-to-road contact for handling and safety. While traditional passive suspension systems can negotiate this trade-off effectively, active suspension systems have the potential to improve both ride quality and handling performance, with the important benefits of better braking and cornering because of reduced weight transfer. This improvement is conditional upon the use of feedback control of the actuators in the active suspension system.

2. Modelling – quarter car
The quarter car model is set up using interconnections of masses, springs and dampers. Figures 1 and 2 show a passive and active quarter car suspension system model, respectively. In these diagrams, $M_s$ is the mass of the car body, $M_{us}$ is the unsprung mass of the wheel and axle assembly, $k_s$ is the spring constant in the suspension system, $b_s$ is the dashpot constant (representing the shock absorber) and $k_t$ is the spring constant of the tyre (the values of these parameters are taken from [1]); $r$ represents the road input force, $x_s$ represents the force acting on the mass $M_s$, $x_{us}$ represents the force acting on the mass $M_{us}$ and $f$ represents the control element (Figure 2 only).

For the passive quarter car model, the state equations may be deduced to be:

\[ M_s \ddot{x}_s = -k_s (x_s - x_{us}) - b_s (\dot{x}_s - \dot{x}_{us}) \]
\[ M_{us} \ddot{x}_{us} = k_s (x_s - x_{us}) + b_s (\dot{x}_s - \dot{x}_{us}) - k_t (x_{us} - r) \]

These equations give the following transfer function relating $x_s$ to $r$:

\[ \frac{x_s}{r} = \frac{k_t (b_s + k_t)}{M_s M_{us} s^4 + (M_s + M_{us}) b_s s^3 + (M_s + M_{us}) k_s + M_t k_t} s^2 + b_t k_s s + b_t k_t \]

For the active quarter car model, the state equations are:

\[ M_s \ddot{x}_s = -k_s (x_s - x_{us}) - b_s (\dot{x}_s - \dot{f}) \]
\[ M_{us} \ddot{x}_{us} = k_s (x_s - x_{us}) + b_s (f - \dot{x}_{us}) - k_t (x_{us} - r) \]
These equations give the following transfer function relating \( x_s \) to \( r \) and \( f \):

\[
x_s = \frac{k_s k_i}{M_s M_w s^4 + (M_s + M_w) b_s s^3 + ((M_s + M_w) k_i + b_s^2 + M_s k_i) s^2 + (2k_s + k_i) b_s s + k_s k_i} r
\]

\[
f = \frac{M_w b_s s^2 + b_s^2 s + (2k_s + k_i) b_s}{M_s M_w s^4 + (M_s + M_w) b_s s^3 + ((M_s + M_w) k_i + b_s^2 + M_s k_i) s^2 + (2k_s + k_i) b_s s + k_s k_i}
\]  \hspace{1cm} (6)

3. Modelling – full car

Figures 3 and 4 show two passive full car suspension system models, with Figure 4 including the effects of the pitch and roll motions.

In these diagrams, the coefficient labels mirror those of Figures 1 and 2; for Figure 4, \( \theta \) represents the pitch angle of the car and \( \phi \) represents the roll angle of the car. The values of the parameters are taken from [2]. For Figure 3, the state equations may be deduced to be

\[
\dot{x}_2 = \frac{1}{M_s} \left[ -(2k_{sf} + 2k_{sa}) x_1 - (2b_{sf} + 2b_{sa}) x_2 + k_{sf} x_3 + b_{sf} x_4 \\
+ k_{sa} x_5 + b_{sa} x_6 + k_{uw} x_7 + b_{uw} x_8 + k_{ur} x_9 + b_{ur} x_{10} \right]
\]

\[
\dot{x}_4 = \frac{1}{M_{us}} \left[ k_{sf} x_1 + b_{sf} x_2 - (k_{sf} + k_s) x_3 - b_{sf} x_4 + k_i r_{fr} \right]
\]

\[
\dot{x}_6 = \frac{1}{M_{us}} \left[ k_{sf} x_1 + b_{sf} x_2 - (k_{sf} + k_i) x_3 - b_{sf} x_6 + k_i r_{fl} \right]
\]

\[
\dot{x}_8 = \frac{1}{M_{us}} \left[ k_{uw} x_1 + b_{uw} x_2 - (k_{uw} + k_s) x_7 - b_{uw} x_8 + k_i r_{fr} \right]
\]

\[
\dot{x}_{10} = \frac{1}{M_{us}} \left[ k_{ur} x_1 + b_{ur} x_2 - (k_{ur} + k_i) x_9 - b_{ur} x_{10} + k_i r_{fl} \right],
\]  \hspace{1cm} (7)

with \( \dot{x}_1 = x_2, \dot{x}_3 = x_4, \dot{x}_5 = x_6, \dot{x}_7 = x_8, \) and \( \dot{x}_9 = x_{10}. \) To develop these equations, \( k_{sf} = k_{sf} = k_{sf}, \) \( k_{sa} = k_{sa} = k_{sa}, \) \( b_{sf} = b_{sf} = b_{sf} \) and \( b_{sa} = b_{sa} = b_{sa}. \) The state variables are assigned as follows:

<table>
<thead>
<tr>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
<th>( x_5 )</th>
<th>( x_6 )</th>
<th>( x_7 )</th>
<th>( x_8 )</th>
<th>( x_9 )</th>
<th>( x_{10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_s )</td>
<td>( z_s )</td>
<td>( z_s )</td>
<td>( z_u )</td>
<td>( z_u )</td>
<td>( z_u )</td>
<td>( z_u )</td>
<td>( z_u )</td>
<td>( z_u )</td>
<td>( z_u )</td>
</tr>
</tbody>
</table>
More involved state space equations may be deduced for Figure 4. Similarly, Figures 5 and 6 show two active full car suspension system models, with Figure 6 including the effects of the pitch and roll motions. The coefficient labels mirror those of Figures 1 to 4.

For Figure 5, the state equations may be deduced to be

\[
\begin{align*}
\dot{x}_2 &= \frac{1}{M_s} \left[-(2k_{d_f} + 2k_u)x_1 - (2b_{d_f} + 2b_u)x_2 + k_{d_f}x_3 + k_{d_f}x_5 \right] \\
& \quad + k_u x_7 + k_u x_9 + b_{d_f}f_n + b_{d_f}f_n + b_{d_f} f_n + b_{u} f_n \\
\dot{x}_4 &= \frac{1}{M_{us}} \left[ k_{d_f} x_1 + b_{d_f} f_n - (k_{d_f} + k_1)x_3 - b_{d_f}x_4 + k_1 r_n \right] \\
\dot{x}_6 &= \frac{1}{M_{us}} \left[ k_{d_f} x_1 + b_{d_f} f_n - (k_{d_f} + k_1)x_3 - b_{d_f}x_6 + k_1 r_n \right] \\
\dot{x}_8 &= \frac{1}{M_{us}} \left[ k_{d_f} x_1 + b_{d_f} f_n - (k_{d_f} + k_1)x_3 - b_{d_f}x_8 + k_1 r_n \right] \\
\dot{x}_{10} &= \frac{1}{M_{us}} \left[ k_{d_f} x_1 + b_{d_f} r_n - (k_{d_f} + k_1)x_9 - b_{d_f}x_{10} + k_1 r_n \right] \\
\end{align*}
\]

with \(x_1 = x_2, \ x_3 = x_4, \ x_4 = x_6, \ x_6 = x_8\) and \(x_9 = x_{10}\). The state equations are developed in a corresponding manner to that of Figure 3.

4. Controller design – active suspension system

An indicative result of simulation work that evaluates the performance of the active suspension system when the feedback controllers are designed using a number of techniques, using the performance of the passive suspension system as a benchmark, is now reported; the simulation work is carried out using MATLAB/SIMULINK. A road disturbance (e.g. a pothole) is simulated; recovery from the disturbance is plotted for the passive suspension system, and the active suspension system when the controller is implemented as a proportional (P) controller or a proportional-integral (PI) controller. The controllers are designed using a standard root locus technique to achieve a +/-2% settling time of one second. Figure 7 shows the simulated disturbance responses.
The results show that the active suspension system allows significant improvement over the passive system. Further such results will be discussed at the symposium.

5. Conclusions
The paper reports on the modelling of passive and active suspension systems, for both a quarter car and full car model. It is shown that the active suspension system facilitates significantly improved regulator response when compared to the passive suspension system. The controlling element of the active suspension system is generally based on an actuator; the main practical difficulty in implementing active suspension is the power consumption of the actuator.

References