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Theoretical Investigations of Electro-optical Synchonisation of Self-pulsating Laser Diodes

A. Egan

G. O'Gorman

P. Rees

Gerald Farrell
Dublin Institute of Technology, gerald.farrell@dit.ie

Joe Hegarty

See next page for additional authors

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Authors

A. Egan, G. O'Gorman, P. Rees, Gerald Farrell, Joe Hegarty, and P. Phelan

Theoretical investigations of electro-optical synchronisation of self-pulsating laser diodes

A. Egan
P. Rees
J. O'Gorman
M. Harley-Stead
G. Farrell
J. Hegarty
P. Phelan

Abstract: The synchronisation mechanism of self-pulsating laser diodes is determined and it is shown how this results in a frequency-dependent phase difference between the electrical input and optical output. The time-to-lock of the electrical input to the optical output is examined, particularly with regard to the importance of the initial phase difference between the signals. The effect of synchronisation on timing jitter in self-pulsating lasers is also investigated.

1 Introduction

The demand for higher speed in communications in recent years, and the anticipated development of optically transparent systems, has led to research into optically based alternatives for certain functions within such systems that hitherto have been carried out electronically. The ultimate goal of this research is the development of an all-optical, transparent, data communications network, although intermediate developments may include certain functions which are executed electro-optically. In the implementation of these alternatives, researchers seek to overcome the limitations of electronics while exploiting the benefits of optics [1]. Self-pulsating laser diodes (SPLDs) have recently attracted attention as devices which may be used to provide certain electro-optical or all-optical functions for optical communication networks; functions such as clock extraction, clock distribution and frequency multiplication or division ([1–4] and references therein).

Self pulsation in the output of semiconductor laser diodes was observed soon after their initial development. The phenomenon was associated with defects,

particularly with age-induced defects [5]. The effect was considered to be undesirable for two reasons; it led to an unwanted variation in the power output of the lasers and it was indicative of degradation in the devices. The mechanism of self-pulsation was later understood to be as a result of saturable absorption owing to defects forming absorbing centres in the lasers [6]. As the quality of devices improved, the phenomenon became less common. In time, as applications for SPLDs were proposed, regions of saturable absorption were deliberately engineered into LDs to produce such devices. One such method involves using a two-section laser diode, with one section biased above threshold (the gain section) and one section biased below threshold (the saturable absorber section) [7, 8]. The repetition frequency of the self-pulsation ν_{SP} can be controlled by varying the bias applied to either section.

For the usefulness of SPLDs in optical communication systems to be fully realised, a greater understanding of the physics of self-pulsation in these devices is necessary, and also of the mechanism of SP synchronisation to electrically and optically injected signals. In this paper, we consider some aspects of electro-optical synchronisation. We present theoretical and experimental results on such issues as the time-to-lock during synchronisation, the importance of the initial phase difference between the SPLD optical output and the applied electrical synchronising signal, the nature of the phase relationship between the two signals when synchronisation has been achieved, and finally the behaviour of pulse jitter during synchronisation. We also provide an explanation of the mechanism of electro-optical synchronisation.

2 The model

For certain problems, the model developed can also be adapted and applied to the behaviour of self-pulsating GaAs (compact disc) laser diodes [9]. In the model for a two-section laser, the gain section extends along a fraction f_g of the laser's length and the absorber section extends along the remaining fraction f_a of the cavity. The photon and carrier dynamics of a two-section SPLD can be described using three rate equations [10, 11] in a single-mode rate equation laser model:

$$\frac{dn_g}{dt} = \frac{j_g}{ed} - \frac{n_g}{\tau_g^c(n_g)} - vg(n_g)S_g \tag{1}$$

$$\frac{dn_a}{dt} = \frac{j_a}{ed} - \frac{n_a}{\tau_a^c(n_a)} + v\alpha(n_a)S_a \tag{2}$$

$$\frac{dS}{dt} = v[\Gamma(f_g g(n_g) - f_a \alpha(n_a)) - \alpha_0]S + \beta B n_g^2 + F_S(t) \tag{3}$$

Eqns. 1 and 2 describe the carrier dynamics in the gain and absorber sections of the laser, respectively. Eqn. 3 describes the photon dynamics in the whole cavity. The symbols are defined as follows (where the subscripts g and a refer to the gain section and absorber section, respectively): $n_{g,a}$ is carrier density, $j_{g,a}$ is DC density, e is electronic charge, d is active region thickness, $\tau_{g,a}^c$ $(n_{g,a})$ is carrier lifetime, v is group velocity of light in the cavity, $S_{g,a}$ is the photon number in each section, S is mean photon number in the whole laser cavity, g is gain (in the gain section), α is loss (in the absorber section), α_0 is the combination of the mirror losses and the internal losses in the laser cavity, β is the fraction of spontaneous emission coupled into the lasing mode, B is the bimolecular radiative coefficient, and $F_S(t)$ is a Langevin noise source which represents spontaneous emission noise. The carrier lifetime in each section may be written as

$$\tau_{a,g}^c(n_{g,a}) = \frac{1}{A_{nr}^{g,a} + B n_{g,a} + C n_{g,a}^2} \tag{4}$$

where A_{nr}^{ga} is the nonradiative recombination coefficient in each section and C is the coefficient of Auger recombination. Assuming a linear approximation for the gain and absorption these terms may be written as:

$$g(n_g) = \frac{a_g(n_g - n_{0g})}{1 + \varepsilon_g S_g} \tag{5}$$

$$\alpha(n_a) = \frac{a_a(n_a - n_{0a})}{1 + \varepsilon_a S_a} \tag{6}$$

where $a_{g,a}$ is the differential gain parameter, $n_{0g,0a}$ is the transparency carrier density, and $\varepsilon_{g,a}$ is the gain/absorption saturation parameter. The photon number in the cavity is a weighted average of the photon number in each section of the cavity and takes into account the fractional lengths of the two sections and the gain and loss in each. The mean photon number is related to the photon number in each cavity by the following equations:

$$S_a = \frac{S}{f_a\left(1 + \dfrac{\gamma_a[(1 - e^{-\gamma_g L f_g})(e^{\gamma_a L f_a} + R_2 e^{\gamma_g L f_g - \gamma_a L f_a})]}{\gamma_g(e^{\gamma_a L f_a} - 1)(1 + R_2 e^{-\gamma_a L f_a})}\right)} \tag{7}$$

$$S_g = S_a \frac{\gamma_a f_a[(1 - e^{-\gamma_g L f_g})(e^{\gamma_a L f_a} + R_2 e^{\gamma_g L f_g - \gamma_a L f_a})]}{\gamma_g f_g(e^{\gamma_a L f_a} - 1)(1 + R_2 e^{-\gamma_a L f_a})} \tag{8}$$

where R_2 is the reflectivity of facet 2, L is the total length of the laser cavity and γ_g and γ_a are:

$$\gamma_g = \Gamma g(n_g) - \alpha_0 \tag{9}$$

$$\gamma_a = \Gamma \alpha(n_a) + \alpha_0 \tag{10}$$

The Langevin noise term $F_S(t)$ is

$$F_S(t) = x\sqrt{\frac{2\beta f_g B n_g^2 S}{\Delta t}} \tag{11}$$

where x is a Gaussian random variable, with zero mean and unity standard deviation, and Δt is the time step in the integration routine used to evaluate eqns. 1, 2 and 3. Note that the noise term depends only on the level of spontaneous emission in the gain section and any contribution from the absorber section is neglected (since the carrier density in this section is always much lower

than that in the gain section). It is well known that noise terms associated with the carrier density are less important for fluctuational properties of the laser and so these terms are neglected here, as in other work of this kind [10]. In the results which follow the parameters used in the modelling are taken from [10] except for those contained in Table 1. This is either because the relevant parameters are not specified in [10] or because they more accurately describe the SPLD used in our experiments.

Table 1: Parameters used in simulations

Parameter	Parameter symbol	Value
Refractive index	n	4.0
Coefficient of Auger recombination	C	$7\times10^{-41}\,\text{m}^6\text{s}^{-1}$
Gain section fraction	f_g	0.8
Absorber section fraction	f_a	0.2
Internal loss	α_i	$1000\,\text{m}^{-1}$
Active region thickness	d	$6\times10^{-8}\,\text{m}$
Wavelength of peak laser emission	λ	$1.5\times10^{-6}\,\text{m}$

3 Time-to-lock

When a periodic electrical signal is applied to a free-running SPLD at a frequency of v_{APP}, which is close to the free-running SP frequency v_{SP}, then, if the power in the electrical signal is sufficient and the difference in frequency is not too great, synchronisation will take place. The self-pulsation frequency of the laser changes from v_{SP} to v_{APP}, and tracks any change in v_{APP}. However, this change does not take place instantaneously, but over a number of pulse cycles. Using eqns. 1–6, we have carried out simulations of the frequency locking or synchronisation process in SPLDs to investigate the time-to-lock and to determine the factors on which this time depends. In this calculation, we do not include spontaneous emission noise in eqn. 3 (as given by eqn. 11). The inclusion of the noise term has little impact on the issues to be discussed at this time and the effect of this noise term on the synchronisation process is considered in a later Section.
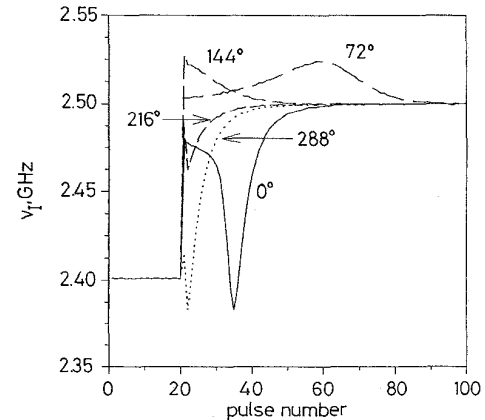
Fig. 1 *Evolution of instantaneous frequency v_I (inverse time between successive pulses) of a SPLD during synchronisation to applied electrical signal of $v_{APP} = 2.5$ GHz with no spontaneous emission noise*
Labels signify the initial phase difference $\Delta\Phi_{DI}$ between applied signal and optical output

Fig. 1 shows the result of a typical simulation. A periodic electrical signal (a pulse with a duty cycle of

10%) with a repetition frequency of $v_{APP} = 2.5\,\mathrm{GHz}$ is applied to the gain section of a laser diode which self pulsates at a frequency of $v_{SP} \simeq 2.4\,\mathrm{GHz}$. The initial phase difference $\Delta\Phi_{DI}$ between the electrical input and the optical output is defined as the difference between the peak of the output optical pulse and the rising edge of the applied electrical pulse. (The electrical pulse leads the optical pulse). Pulse number refers to the optical train pulse number. For clarity, we characterise the locking in terms of an 'instantaneous frequency' v_I which is the inverse of the time interval between successive optical pulses. The behaviour of v_I as a function of pulse number is determined for a number of values of $\Delta\Phi_{DI}$. The electrical signal is applied at pulse 20 and one sees that, although synchronisation does eventually take place for each value of $\Delta\Phi_{DI}$, the time required for v_{SP} to be pulled to v_{APP} is different in each case. It ranges from about 20 pulse cycles in the case of $\Delta\Phi_{DI} = 216°$ to about 90 pulse cycles in the case of $\Delta\Phi_{DI} = 72°$. This Figure suggests that the time-to-lock depends critically on the initial phase difference between the electrical input and optical output. To understand the reason for this dependence, we needed first to understand the mechanism of electro-optical synchronisation.

4 Mechanism of electro-optical synchronisation

Synchronisation of the optical frequency from a SPLD to an electrically injected periodic signal takes place due to the perturbation of the laser carrier density by carriers injected by the synchronising signal. Synchronisation occurs because these extra carriers have the effect of changing the time at which subsequent optical pulses are emitted from the laser. The free-running SP frequency of the laser may initially be lower or higher than v_{APP}; therefore these two possibilities lead to two different cases for synchronisation, which are now dealt with in turn.
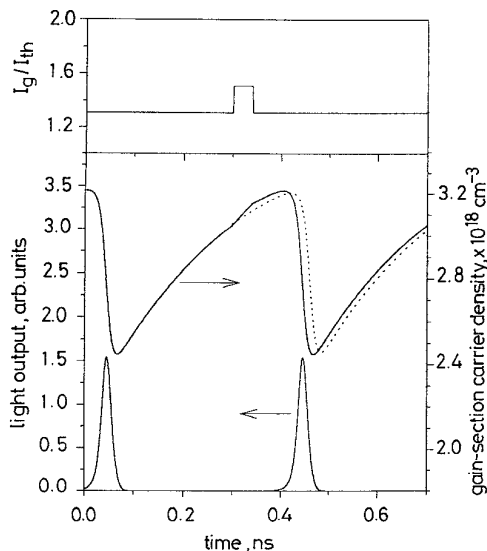

Fig.2 *Temporal evolution of normalised gain-section current density I_g/I_{th} (I_{th} = threshold current), light output and gain-section carrier density, after synchronisation of optical output from SPLD to electrically injected periodic signal when $v_{APP} = 2.5$ and $v_{SP} = 2.4\,GHz$*

Fig. 2 corresponds to electro-optical synchronisation when $v_{APP} > v_{SP}$. A periodic electrical pulse-like signal of frequency $v_{APP} = 2.5\,\mathrm{GHz}$ and duty cycle of 10% is applied (through the gain section) to a free-running self-pulsating laser with $v_{SP} = 2.4\,\mathrm{GHz}$. In the Figure,

synchronisation has taken place and the temporal profiles of the electrical input to the laser, the optical output and the carrier density in the gain section are shown. The electrical pulse modifies the carrier density evolution in the gain section somewhat before the emission of an optical pulse. For the duration of the electrical pulse, the rate at which the carrier density increases is higher than it would otherwise be, and this hastens the emission of the next output pulse. In Fig. 2, the dotted line represents the approximate evolution of the gain section carrier density in time if the electrical pulse had not been applied to the laser. It is clear that the threshold level of carrier density at the emission of the subsequent pulse would have been reached at a later time, had it not been for the applied electrical pulse. Hence, the period between the optical pulses decreases and the SP frequency increases from v_{SP} to v_{APP}. Synchronisation occurs because the next optical pulse is hastened by just the right amount to ensure that the next electrical pulse interacts with the carrier density at exactly the same level as before, and so the cycle continues.
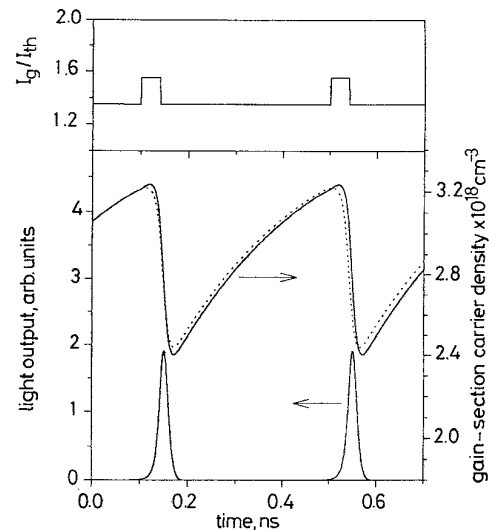

Fig.3 *Temporal evolution of normalised gain-section current density I_g/I_{th}, light output and gain-section carrier density after synchronisation of optical output from SPLD to electrically injected periodic signal when $v_{APP} = 2.5$ and $v_{SP} = 2.53\,GHz$*

Electro-optical synchronisation for the case of $v_{APP} < v_{SP}$ is indicated in Fig. 3. The electrical input is the same as that described in Fig. 2, but this time $v_{SP} = 2.53\,\mathrm{GHz}$. The interaction of the electrical pulse with the gain section carrier density roughly coincides with the presence of an optical pulse in the cavity. Again, the dotted line in Fig. 3 represents the approximate path of the carrier density in time, if the electrical pulse had not been injected into the cavity. The presence of the electrical pulse leads to an appreciable extra depletion of the carrier density during the emission of the optical pulse. When the carrier density begins to increase again, it does so from a lower level than would have been the case in the absence of the injected electrical pulse. This means that the threshold condition for the emission of the next optical pulse is reached at a later time than would otherwise have been the case (compare the solid line and dotted line for the carrier density at the time of emission of the second optical pulse in Fig. 3). In this way, the period between the pulses is increased and the SP frequency decreases from v_{SP} to v_{APP}.

An important point to note from Figs. 2 and 3 is that the phase difference between the electrically injected pulses and the optically emitted pulses is different in the two cases. The temporal profile of the carrier density between optical pulses is nonlinear, so that the efficiency with which the electrical pulse causes the gain section carrier density to increase depends on the time at which it interacts. In other words, the effect of the input electrical pulse depends on the phase difference between it and the preceding optical pulse. Obviously, the converse of this is also true: the phase difference between the electrical and optical pulse defines the effect which the electrical pulse has. This explains the sensitivity of the time-to-lock to the initial phase difference between the signals (as shown in Fig. 1). When the signals synchronise, they do so with a fixed and definite phase relationship. For two particular frequencies v_{SP} and v_{APP} there is only one stable phase difference between the signals which will allow synchronisation to be achieved. When an electrical signal is applied to a SP laser at some phase difference other than that which is the final, synchronised phase difference, the phase difference will change to this value over subsequent pulse cycles, both because the electrical signal successively modifies the period between the pulses and also because of the initial frequency difference between the synchronising signal and the SP output. The rate at which this process takes place depends on the initial phase difference between the signals, as well as the relative frequency difference between them.

The final phase difference between the two signals depends on the frequency difference $v_{SP} - v_{APP}$. When a periodic electrical signal of frequency v_{APP} is applied to a SPLD of free-running SP frequency v_{SP} the effect of the input electrical pulses is to alter the period of the output optical pulses from $1/v_{SP}$ to $1/v_{APP}$. The magnitude of the change in the period depends on the frequency difference $v_{SP} - v_{APP}$. But this in turn is defined by the relative phase difference $\Delta\Phi_D$ between the signals when they interact. Consequently $\Delta\Phi_D$ depends on $v_{SP} - v_{APP}$, among other parameters. This results in a relative phase difference between the electrical input and the optical output, as has been shown by Georges et al. [12].
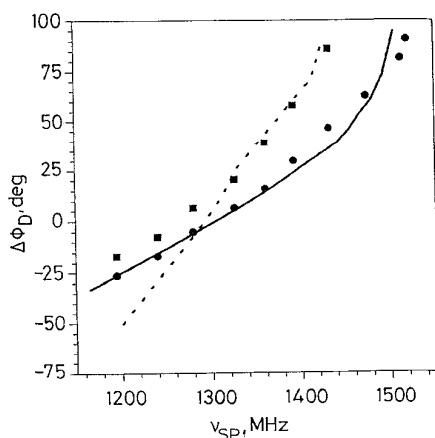

Fig. 4 *Variation in phase difference $\Delta\Phi_D$ between applied electrical signal and laser output pulsation relative to phase difference at $v_{SP} = 1300\,MHz$, as free-running self-pulsation frequency varies*
Phase difference of $\Delta\Phi_D = 0°$ does not necessarily correspond to an absolute phase difference of 0°
● experimental data, large signal level
■ experimental data, low signal level
—— theoretical predictions, large signal level
– – – theoretical predictions, low signal level

We have also carried out experiments to verify our calculations and to demonstrate the frequency dependence of this electro-optical phase difference [13]. The device used in the experiments was a two section Fabry–Perot laser, such as is described in [7], and the model used to simulate the experiments was based on this device. Fig. 4 shows a comparison of calculated and experimental results. In the experiment and simulations, a sinusoidal electrical signal of $v_{APP} = 1300\,MHz$ is applied to a two-section SPLD. In the preceding analysis we have discussed the application of pulse-like electrical signals to lasers, since this localised injection of carriers lends itself more readily to the analysis we have carried out. However, the analysis is equally valid for sinusoidal signals, as is demonstrated in Fig. 4. The data points represented by the circles may be compared with the solid line which has been calculated using the model outlined in Section 2. The total range over which the phase may vary is seen to be 120°. When the power in the synchronising electrical signal is reduced by 3dB the data points represented by the squares and the dashed line result. One sees that $\Delta\Phi_D$ depends also on the power in the applied electrical signal. The agreement is good for frequencies higher than v_{APP} but not so good for lower frequencies. We believe that this is because of poor quality synchronisation at the lower frequencies, owing to the fact that the lower limit of self-pulsation in the device is represented by the lower limit of the data points. Nevertheless, the agreement is very good, especially given the simplicity of the model used.

This phase dependence on the frequency difference between the applied signal and the LD self-pulsation has important consequences for communication systems using SPLDs for clock extraction. The observed frequency dependent nature of the phase shift shows that all synchronised states are not the same, at least as far as phase is concerned. In some cases, this phase variation may be a desirable effect, for example, it may be possible to exploit it for fine tuning the clock delay in clock extraction systems. In other cases, it may be preferable if the effect is minimised. Consequently an understanding of the origin of this phase shift and its behaviour is necessary if advantage is to be taken of it. Such an understanding may allow enhancement or diminishment of the total phase variation over the synchronisation range, according to the requirements of a particular application.

5 Effect of spontaneous emission noise on synchronisation

In Fig. 1, we simulated the electro-optical synchronisation process in a SPLD when noise is not included in the rate equation model; in this Section, we examine the effect of including spontaneous emission noise in the photon rate equation as given by eqns. 3 and 11. Fig. 5 shows the results of these simulations. The dotted lines represent the development of v_I when the noise term is not included in the model and for the two initial phase differences specified in the Figure. These curves correspond exactly to the two curves for the same initial phase differences shown in Fig. 1. The two solid lines represent the development of v_I when noise is included in the model and are an average over 50 synchronisation runs. The average development of v_I with pulse number is similar with and without noise in the simulation. For the conditions pertaining to Fig. 5

synchronisation takes place over about 40 pulse cycles for $\Delta\Phi_{DI} = 0°$, and over about 20 pulse cycles for $\Delta\Phi_{DI} = 216°$. The exact length of time over which this process occurs will always depend on both the frequency difference between the signals and the initial phase difference between them.
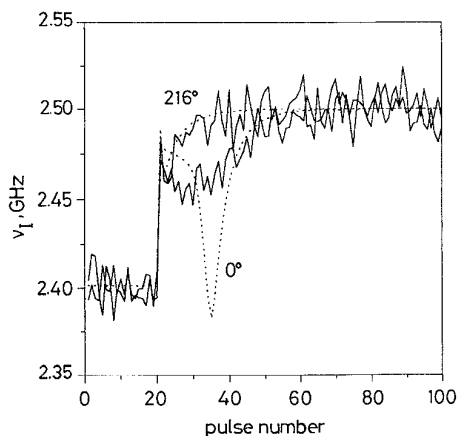

Fig. 5 *Development of v_I of SPLD during synchronisation to applied electrical signal of $v_{APP} = 2.5\,GHz$*
----- No spontaneous emission noise, $\Delta\Phi_{DI} = 0°$ and $216°$
——— Average of 50 synchronisation runs, noise included

Frequency lock-in is just one aspect of electro-optical synchronisation; another important aspect is that of jitter reduction. We have also examined the jitter reduction characteristics of electro-optical synchronisation in SPLDs using our rate equation model. We characterise the jitter in the optical output from a SPLD as the full width at half maximum (FWHM) of the jitter distribution curve, i.e. the curve (or histogram) which defines the distribution in the emission time of the pulses about their mean position. In this way, we characterise the jitter of an optical pulse with the immediately preceding optical pulse (pulse-to-pulse jitter) or with a less recent optical pulse, so that we may compare the short-term, pulse-to-pulse jitter with the longer term jitter (over a few pulses).
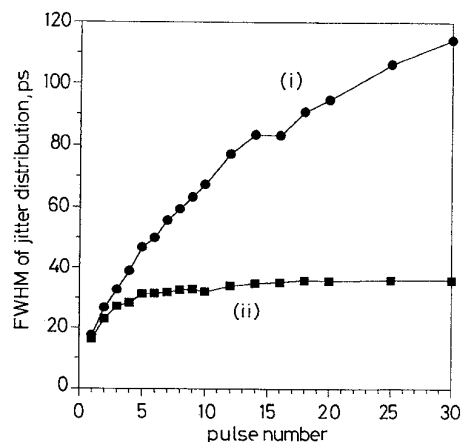

Fig. 6 *FWHM of jitter distribution of pulse number n with respect to zeroth pulse*
Free-running SP frequency $v_{sp} = 2.5\,GHz$
(i) unsynchronised (no electrical injection)
(ii) synchronised
● Simulation of SPLD, no injection
■ Simulation of SPLD, sinusoidal injection at $v_{APP} = 2.5\,GHz$

In Fig. 6 we have plotted the FWHM of the jitter distribution curves for one, two, three, etc. pulse separations, for the case of a SPLD which is free-running and for the case when it is synchronised to an electrical

pulse-like signal with $v_{APP} = 2.5\,GHz$. For pulse number 1 (i.e for pulse-to-pulse jitter) the magnitude of the jitter is about the same for the synchronised laser as for the free-running laser. However, as pulse number increases, the jitter in the free-running laser continually increases for the range shown in the Figure. On the other hand, the plot for the free-running laser shows that, after about five pulses, the magnitude of the pulse jitter saturates (at about twice the value of the pulse-to-pulse jitter). Fig. 6 demonstrates that synchronisation of a SPLD to an applied electrical signal is not a process in which the position of every pulse in time is exactly fixed by the injected signal, but rather the mechanism of synchronisation leads to the ability of the applied signal to prevent the accumulation of timing jitter.

In summary, jitter reduction manifests itself over a range of about five pulses. Frequency lock-in, on the other hand, may take much longer. For the example we have provided, it was seen to take between about 20 and 40 pulses. Clearly then, knowledge or control of factors such as the initial phase difference or the relative frequency difference $v_{SP} - v_{APP}$ may permit minimisation of the time-to-lock in a particular application involving electro-optical synchronisation. Time-to-lock is an important parameter in communications systems since it will dictate the duration between the reception of the first data bit and the time at which an extracted clock can be reliably used. In a switched communications environment a long time-to-lock would be inefficient since a significant number of redundant data bits would be required as a preamble in each data signal. In addition, the presence of a fixed level of jitter will result in a bit error rate penalty for systems which utilise an optical clock generated electro-optically by a self-pulsating laser diode.

6 Conclusions

We have investigated the nature of electro-optical synchronisation in self-pulsating laser diodes. A rate equation model has been outlined and results which have been obtained using this model have been presented. We have provided an explanation of the mechanism of electro-optical synchronisation, particularly with regard to the frequency pulling aspects of synchronisation, when the repetition frequency of the injected electrical signal is greater than or less than the free-running self-pulsation frequency of the laser. We have experimentally and theoretically shown that, when synchronisation takes place, the signals have a definite phase relationship with one another which depends on the difference between the free-running SP frequency and the frequency of the injected signal, and on the power in the synchronising signal.

We have investigated the time-to-lock associated with electro-optical synchronisation, where synchronisation is seen to be composed of two aspects; frequency lock-in and jitter reduction. We have seen that simulations show that jitter reduction occurs over the space of about five pulses and frequency lock-in typically takes place over a greater number of pulses. The duration required for frequency lock-in depends on the relative difference between the free-running SP frequency and the frequency of the input signal and on the initial phase difference between the signals. Such considerations have importance in the design of electro-optical clock extraction systems. The existence and the extent

of these effects are also important considerations in all-optical communication systems and the subject of ongoing investigations.

7 Acknowledgments

8 References

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