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HYDRO-ENVIRONMENTAL MODELING OF SEWAGE AND RIVERINE DISCHARGES INTO A COASTAL AREA: COMPARISON OF DEPTH-AVERAGED AND THREE-DIMENSIONAL MODELS

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Abstract

This study applies and compares two hydrodynamic and water quality models; a depth-averaged (TELEMAC-2D) and a three-dimensional model (TELEMAC-3D) on their performance in simulating the transport and fate of *Escherichia coli* (a main microbial bathing water quality indicator) in the coastal waters of Bray, Ireland subjected to sewage discharges and freshwater inflows from the River Dargle. The models first calibrated and validated against hydrodynamic and water quality data, were used to simulate *Escherichia coli* distribution patterns based on mean spring and mean neap tides for dry and wet weather scenarios. The hydrodynamic calibration yielded a good match between both models (TELEMAC-2D and TELEMAC-3D) and measured velocities. The *E. coli* model calibrations showed that TELEMAC-2D resulted in a lower value for decay rate (higher T_{90} value) than TELEMAC-3D in order to match the measured *E. coli* concentrations. *E. coli* surface distributions at the time of HW resulted in TELEMAC-2D plumes that were lesser in extent and concentrations than those of TELEMAC-3D due to the fact that depth-averaged hydrodynamics underestimate the surface water velocity resulting in lower concentrations of *E. coli* at the water surface compared to TELEMAC-3D. The wet weather scenarios of both TELEMAC-2D and TELEMAC-3D exhibited high *E. coli* concentrations at the water surface that exceed the "Sufficient" limit of the Bathing Water Directive, the latter finding highlights the need for including Ultra Violet disinfection in the treatment process at Shanganagh Sewage Treatment Works.

Keywords: Water quality modelling; three-dimensional; depth-averaged; sewage discharges; Bathing Water Directive

1. Introduction

Achieving and maintaining high water quality standards for marine waters is an essential requirement for supporting the various activities and demands on coastal areas. Such a requirement has been enforced by growing and more stringent EU environmental legislation such as the revised Bathing Water Directive 2006/7/EC (EC, 2006) which sets strict standards for the microbiological quality of bathing waters.

In Ireland, Bray beach, Co. Wicklow is of a high recreational and heritage value and is a designated EU bathing site, serving a rapidly growing population at Bray town as well as nearby towns and villages. The beach receives inland flow from the upland forests and

agricultural Dargle catchment via the Dargle River and tributary network. The catchment is 'flashy' and short, thus intense rainfall events in the upland catchment produce runoff that is the main source of episodic short-term pollution incidents of the near-shore coastal waters of Bray beach. Another pressure on the bathing water quality at Bray beach is sewage discharges. Until recently, untreated wastewater from Bray town was discharged through a long-sea outfall (approximately 1.5km offshore Bray beach). In 2012 a major upgrade was made to the Shanganagh Sewage Treatment Works (STW) in Figure 1 to improve the quality of discharged sewage. Although these works have been recently completed and their benefits remain to be assessed, previous water quality investigations in the River Dargle catchment indicate that rainfall-related runoff can seriously impact the water quality of the near-shore marine waters (Bruen et al., 2001). The river/coastal interactions of the system where the River Dargle discharges to the Irish sea is therefore complex and may be best assessed through numerical modelling approaches.

This is in keeping with the increasing use of numerical models as tools to better understand environmental systems and predict their responses to pollution incidents. Typically, they solve a set of governing, physically based equations describing the flow and the transport of contaminants. The accuracy of the solution depends on how adequately these equations reflect the actual physical conditions. In practice, two different types of models have evolved – depth-averaged and three-dimensional – each representing a different compromise between ease of use (depth-averaged models) and better representation of the spatial aspects of the behavior (three-dimensional models). Depth-averaged models such as DIVAST (Falconer, 1986), MIKE21 (DHI, 2011), and TELEMAC-2D (Hervouet, 2007) integrate hydrodynamic and/ or water quality variables over a vertical water column and thus neglect variations in density and contaminant concentrations over the water column. On the other hand, three-dimensional models for example TRIVAST (Binliang and Falconer, 1996), EFDC (Hamrick, 1992), TIDE3D (Walters, 1987) and TELEMAC-3D (Hervouet, 2007) solve the Navier-Stokes set of equations. Most of these models simulate the mass transport of active tracers (i.e. tracers that influence water density such as temperature, salinity and sediments) and incorporate their effect on the flow hydrodynamics. This feature favors the use of such models in stratified environments (see for example Bedri et al. 2013; Ji, et al., 2007; Chao et al, 2008).

The presented work applies and compares two hydrodynamic and water quality models; a depth-averaged (TELEMAC-2D) and a three-dimensional model (TELEMAC-3D) on their performance in simulating the transport and fate of *Escherichia coli* (a main microbial bathing water quality indicator) in the coastal waters of Bray subjected to sewage discharges and catchment runoff. The models are first calibrated and validated against hydrodynamic and water quality data. They are then used to simulate *Escherichia coli* distribution patterns based on a number of weather scenarios.

2. Study Area

The coastal area under study is located on the east coast of Ireland (Longitude 6.1°W, Latitude 53.22 °N). It is bounded in the north and the south by Dalkey and Bray headlands and stretches approximately 10 km in length (Figure 1). The land slopes gradually in the seaward direction (to the East) from low water to a depth of 8 m after which it falls more steeply to reach 20-25m at a distance approximately 2 km offshore. Observations on tidal streams show that a flood tide enters the study region strongly from the south, but is deflected by Bray Head and runs parallel to the coastline before being deflected outwards by Dalkey Headland. The southwards

flowing ebb tide also runs parallel to the coastline and serves to draw water out of Bray Harbor. Tidal ranges in this region have a mean range of 2.75m and average mean spring and neap tides of 3.6m and 1.9m respectively (Mansfield, 1992). The Dargle River is the main freshwater inflow into the coastal waters under study. The river draining a catchment of circa 133 km², is characterized by a steep gradients of approximately 2.7%. The Dargle River has an average dry weather discharge of 3 m³/s which may rise to 300 m³/s during extreme flood events.

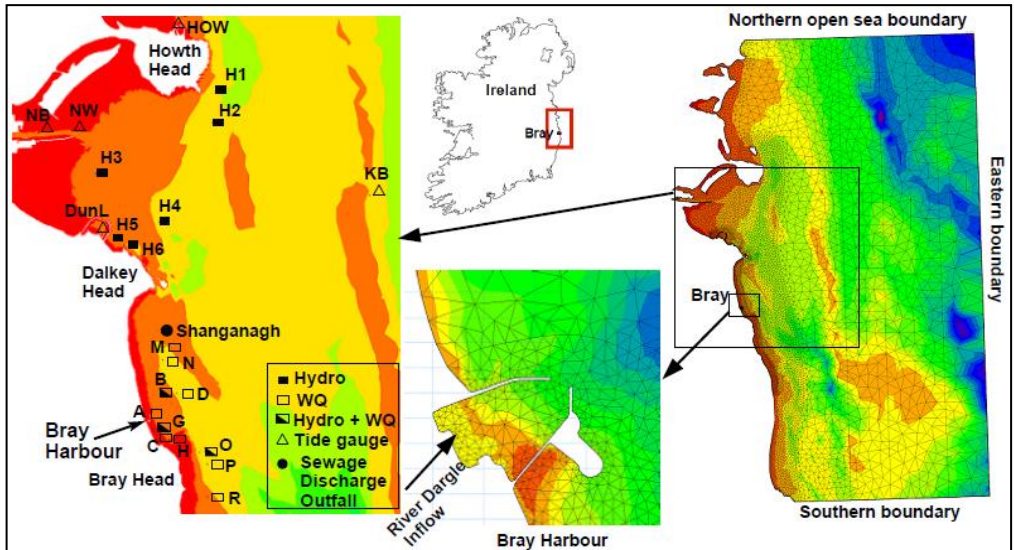


Figure 1. Study area: sampling points in coastal waters (bottom left), TELEMAC mesh (top right), and map Bray Harbor (top left)

3. Methods

3.1 Description of Models

The TELEMAC modelling suite for free-surface flows (Hervouet, 2007), developed by the National Laboratory of Hydraulics and Environment of Electricité de France (EDF), was selected because it is based on a finite element grid which allows the selective refinement of the computational mesh.

The current study applies the hydrostatic version of the three-dimensional model TELEMAC-3D (Hervouet, 2007) which solves the 3D Reynolds-Averaged Navier-Stokes (RANS) momentum and continuity equations in unstructured meshes obtained by a superimposition of 2D meshes of triangles (Villaret, et al. 2013). The model of the current study includes the effect of wind and heat exchange with the atmosphere on flow dynamics. It also incorporates the effect of temperature and salinity on water density, which in turn affects flow hydrodynamics and transport of tracers. The evolution of temperature and salinity in space and time are computed by the TELEMAC-3D tracer mass-balance equation. The current study however, neglects the effect of wave radiation on the flow and transport processes. The spatially and temporally varying concentrations of *E. coli* (the water quality parameter under study) are resolved using the tracer mass-balance equation which in its original form only resolved for

advection and dispersion processes. The equation has been adapted by Bedri et al. (2013) to include a decay function for *E. coli*, based on a formula by Mancini (1978) which incorporates the effect of temperature, salinity, and light penetration on the decay of *E. coli*. For the resolution of the horizontal turbulence, the Smagorinski (1963) sub-grid scale model has been chosen. The scheme is commonly used in maritime domains (see Marques et al. 2009; Hall and Davies, 2007) with large scale eddies where small vortices can be inhibited by the mesh. A Prandtl mixing length scale model with a Mark and Anderson damping function is used for the vertical turbulence closure.

The TELEMAC-2D model solves the depth-averaged Saint-Venant momentum and continuity equations. The depth-averaged tracer mass-balance equation of TELEMAC-2D including a constant die-off function is used to compute the time-varying concentrations of *E. coli* at any point in the model domain. For the resolution of turbulence closure, the Smagorinski (1963) model is also used for the purpose of the comparison with TELEMAC-3D.

3.2 Model set-up and calibration

The coastal model domain extending a distance of 64 km in the north-south direction and 44 km in the east-west direction was discretized using an extensive bathymetric data set to form a finite element mesh of 38853 nodes and 73122 elements that range in size from 25 m to 1.8 km (Figure 1). For the vertical discretization of the TELEMAC-3D model, five boundary fitting (sigma-transformed) layers were used. The domain has three open-sea boundaries; northern, eastern, and southern, located boundaries far beyond the area in interest in order to reduce their influence on the numerical solution. Time- and space- varying water elevations at these boundaries were extracted from the Danish Hydraulic Institute (DHI) Global Tidal database (DHI, 2013). The calibration of the models (TELEMAC-3D and TELEMAC-2D) was completed in a two-step procedure; hydrodynamics then water quality. Firstly, the hydrodynamic component of TELEMAC-2D was set-up using: the mesh, initial conditions (zero water velocities, and a constant mean sea level), and boundary conditions (open-sea tidal elevations at the boundaries, measured flow time-series of the River Dargle, and time-series of wind speed and direction measured by a weather station at Bray Harbor). The set-up of the TELEMAC-3D model required the same set-up variables of TELEMAC-2D in addition to time series of air temperature (obtained for the weather station at Bray Harbor for heat-exchange process), initial and background values of temperature and salinity at both the model boundaries (obtained from water quality surveys) and the River Dargle inflow boundary (obtained from measurements). The hydrodynamic models were calibrated by adjusting the bottom friction parameter (Chezy number) and comparing simulated hydrodynamic variables to measurements. Simulated TELEMAC-3D depth profiles of temperature and salinity at the mouth of Bray Harbor (location A in Figure 1) were also compared to measurements. Thereafter, the water quality components of TELEMAC-2D and TELEMAC-3D were set-up to simulate the transport and fate of *E. coli* in the coastal waters using: (i) measured flow and *E. coli* concentration at the River Dargle outlet into the coastal area, (ii) measured *E. coli* flow and concentrations of the Shanganagh STW, (iii) initial/ background values of *E. coli* - these were set to zero but the model was initially run for a warm-up period of 5 tidal cycles to establish background conditions; and (iv) boundary conditions of *E. coli* - zero concentrations were imposed at the open-sea boundaries which are located far out at sea where the distance from the shoreline is sufficient to negate the effect of boundary conditions on the computations in the area of interest (Bray beach). The water quality component of TELEMAC-3D was calibrated

by varying a parameter ($k_e H$) in the Mancini formula (see Bedri et al., 2013) and comparing simulated E. coli to measurements at the water quality sampling points. The TELEMAC-2D model was calibrated by varying the constant decay parameter.

3.3 Modelling scenarios

Based on the calibrated hydrodynamic and water quality components of TELEMAC-2D and TELEMAC-3D, a number of modelling scenarios were formulated in order to compare and assess the performance of the two models in simulating the distribution of E. coli in the coastal waters under dry and wet weather conditions (Table 1). The simulations were based on two representative tide types; mean spring (MST) and mean neap (MNT) tides.

Table 1: Discharge Scenarios. Q: Discharge (m^3/s), EC: E. coli concentration (cfu/100ml)

Model Scenario	River Dargle		Shanganagh STW	
	Q	EC	Q	EC
Dry weather conditions	2.7	2400	0.45	125000
wet weather conditions	10.21	36200	1.18	1500000

4. Results and Discussions

4.1 Calibration of Hydrodynamic components of TELEMAC-2D and TELEMAC-3D

4.1.1 Velocities

The best match between observed and predicted current velocity was achieved when using a Chezy coefficient of 60, for the bottom friction factor. Simulated current speeds using TELEMAC-3D and TELEMAC-2D (Figure 2) were compared to measurements taken at the water surface and bottom at location O. Both models have replicated well the measured pattern of the flooding tide (period before time of high water (HW)), particularly TELEMAC-3D which gave a better match to the residual currents around the time of low water (HW-6 hrs). TELEMAC-3D showed a good match to the bottom velocity measurements of the flood stage but simulated velocities at the water surface overestimated the first peak by approximately 0.1 m/s while TELEMAC-2D showed a better match to measured velocity at the surface than the bottom. On the ebb stage of the tide, both TELEMAC-2D and TELEMAC-3D gave a reasonable fit to current speed measurements of the surface and bottom speeds for the period HW+4 to HW+6.5 hrs but could not match the measurements of the period (HW to HW+4 hrs) which demonstrated some discrepancies showing bottom measurements greater in magnitude than bottom speeds. Both models have adequately replicated the general pattern of the observed current direction which exhibited a north-westerly flow direction during a flood tide and a south-easterly direction during an ebbing tide. However, they overestimated the flooding tide direction by approximately 20° and underestimated the ebbing tide direction by approximately 30° .

4.1.2 Temperature and salinity

Simulated temperature and salinity profiles using TELEMAC-3D were compared to measurements (shown as points in Fig. 2) taken along the water column at point A, located in the vicinity of the harbor mouth. The simulated temperature profiles demonstrated a generally better fit to measurements than salinity profiles, particularly closer to the water surface where discrepancy between measured and simulated temperature and salinity at $z/h=0.1$ was approximately 0.2°C and 1 PSU respectively. Also the lower half of the water column demonstrated a better fit between simulated and measured temperature and salinity than the upper half (Figure 2).

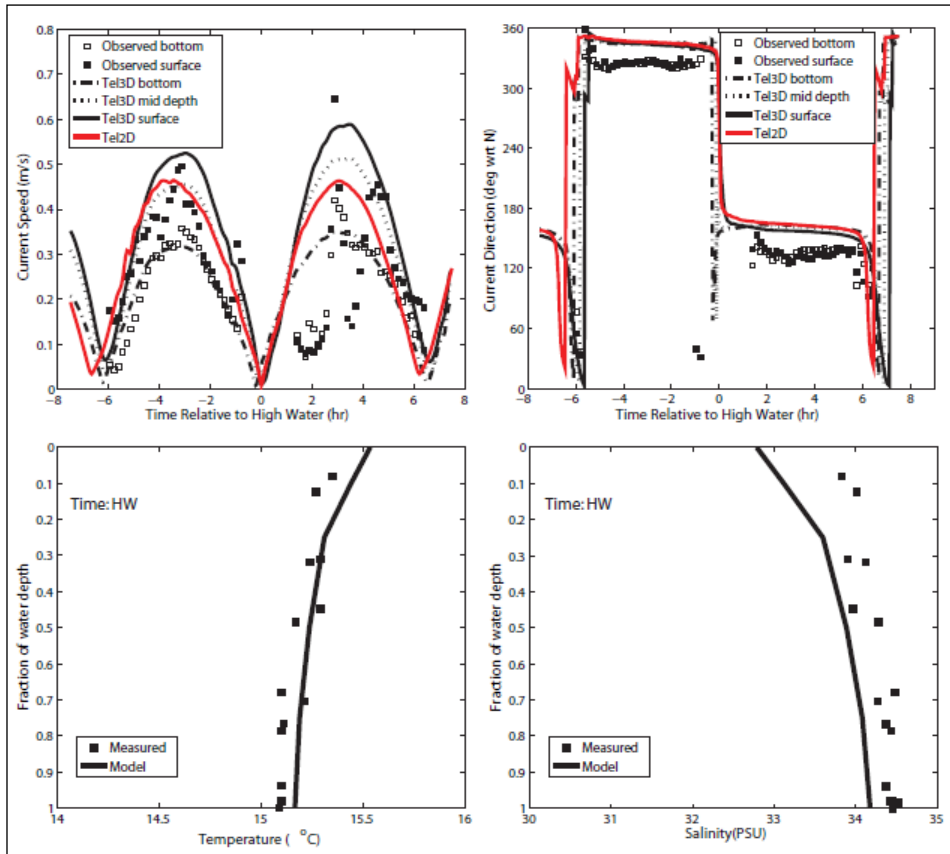


Figure 2: Observed and simulated TELEMAC-2D and TELEMAC-3D current speed and direction at location O (Top) and Observed and simulated (TELEMAC-3D) temperature and salinity depth profiles at time of High Water (HW), location A (bottom)

4.2 Calibration of E. coli model: TELEMAC-2D and TELEMAC-3D

The best fit between surface measurements and predictions of E. coli patterns by TELEMAC-2D was obtained with a decay rate (T_{90}) value of 36 hours while the best fit with TELEMAC-3D was obtained with a value of 0.6 of the $k_e H$ factor. The resulting TELEMAC-3D decay rate corresponded to a T_{90} value of approximately 24 hours. The lower decay rate (higher T_{90} value) required by TELEMAC-2D can be explained by the fact that depth-averaged hydrodynamics do not account for the larger velocities at the water surface as TELEMAC-2D can neither

produce a proper vertical velocity distribution nor can it simulate the effect of temperature and salinity on the water density (and subsequently on flow hydrodynamics). Therefore TELEMAC-2D tends to under-predict the advection of E. coli. This is compensated for in the depth-averaged model by reducing the decay rate. The simulated time series of two points (N and R) indicate a unique pattern at each of the two points with both models TELEMAC-2D and TELEMAC-3D showing a similar pattern over the tidal cycle. At point N, the simulated E. coli concentrations rise quickly from a minimum around the time of low water to reach a peak around the time of mid flood after which they gradually drop again while at point R, the model exhibits a drop in E. coli concentrations during the flooding stage reaching a minimum around the time of high water (Figure 3), then they rise again. Comparison between measured and simulated E. coli concentrations at N shows that both model generally captures the order of magnitude of observed concentrations and match somewhat the observed patterns at mid-flood and around the time of HW but underestimate the observed E. coli concentrations at mid-flood peak. At point R, both models particularly TELEMAC-2D present a good fit to observed E. coli concentrations of the flood stage (time before HW) but both underestimated measured E. coli concentrations around the time of low water (HW+5 to HW+6 hours).

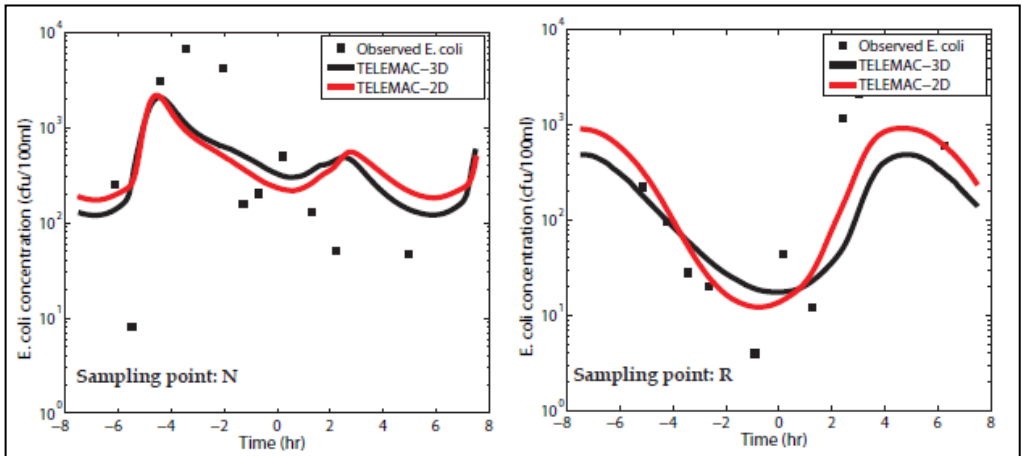


Figure3. Observed and simulated E. coli at locations N and R

4.3 Comparison of E. coli distributions

4.3.1 Dry weather conditions

Figure 4 shows the TELEMAC-3D distribution of E. coli at the water surface and the depth-averaged E. coli distribution of TELEMAC-2D at time of HW during dry weather conditions. Closer to the time of HW, the north-flowing flood currents in the outer sea weaken considerably while they start to deflect southwards in the near-shore area, causing the plume around Bray Harbor, resulting from the River Dargle, to start to flow southwards. A comparison between the two tide types (MST and MNT), show that the MNT results in an E. coli plume of greater concentration due to the lower flux of water compared than that of a MST tide. Figure 4 also demonstrates that the size and concentrations of the E. coli plume at Shanganagh STW simulated by TELEMAC-2D is very much less than that produced by TELEMAC-3D. This is because depth-averaged hydrodynamics tend to underestimate the surface water velocity resulting in lower concentrations of E. coli at the water surface

compared to those of TELEMAC-3D. However, the concentrations of *E. coli* of a dry weather scenario are generally very low, due to the improved level of treatment at Shanganagh STW in addition to the low yield of *E. coli* from the River Dargle during dry weather conditions. This results in *E. coli* concentrations at Bray beach that are well below the “Sufficient” water quality limit of 500 cfu/100ml set by the revised Bathing Water Directive (2006/7/EC).

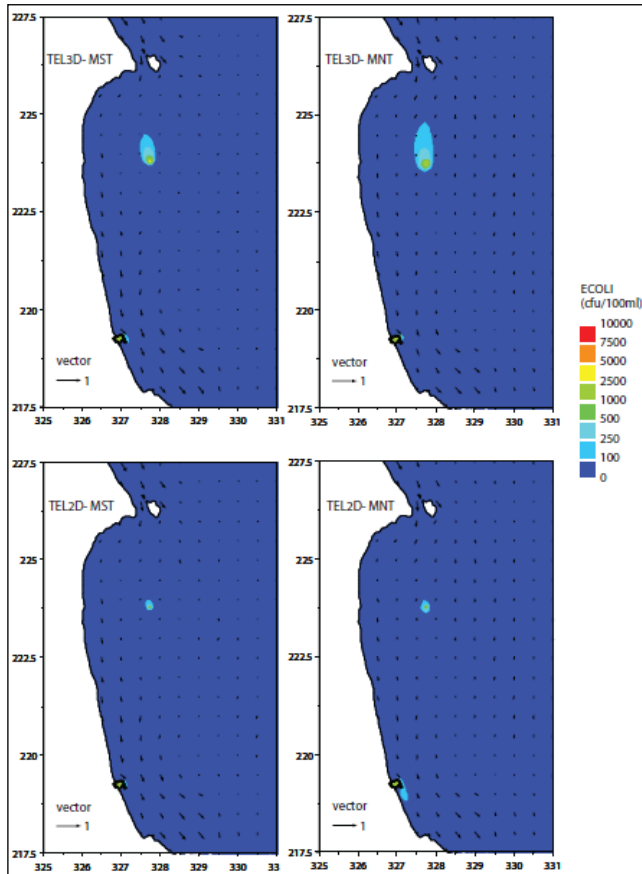


Figure4. Dry weather conditions: *E. coli* distributions of TELEMAC-2D and TELEMAC-3D

4.3.2 Wet weather conditions

The simulated surface distributions of *E. coli* around the time of HW show plumes of much larger extent and concentrations (Figure 5) in comparison to those of dry weather conditions (Figure 4). This can be attributed to the high diffuse loadings from the River Dargle associated with wet weather conditions in addition to the effect of storm overflows at Shanganagh STW. In a similar comparison between MST and MNT in Figure 4, the stronger currents exhibited by a mean spring tide causes the discharged *E. coli* at the outfall to separate and quickly dissipate resulting in a plume of wider extent but lesser concentration than that of a mean neap tide. In a similar observation of the *E. coli* distributions of TELEMAC-2D and TELEMAC-3D in Section

4.3.1, TELEMAC-2D model produced plume of lesser extent and concentrations that those of TELEMAC-3D. Comparison of E. coli distributions of dry and wet weather conditions show extremely high concentrations at Bray beach that are well above the “Sufficient” limit of 500 cfu/100ml of Directive 2006/7/EC. This finding highlights the need for tertiary treatment (Ultra Violet (UV) disinfection) during the Bathing Season (mid May – mid September) in order to reduce the E. coli loading into the marine waters. While UV disinfection is not currently a part of the treatment processes at Shanganagh STW, provisions have been allowed for its inclusion during the recent upgrade works.

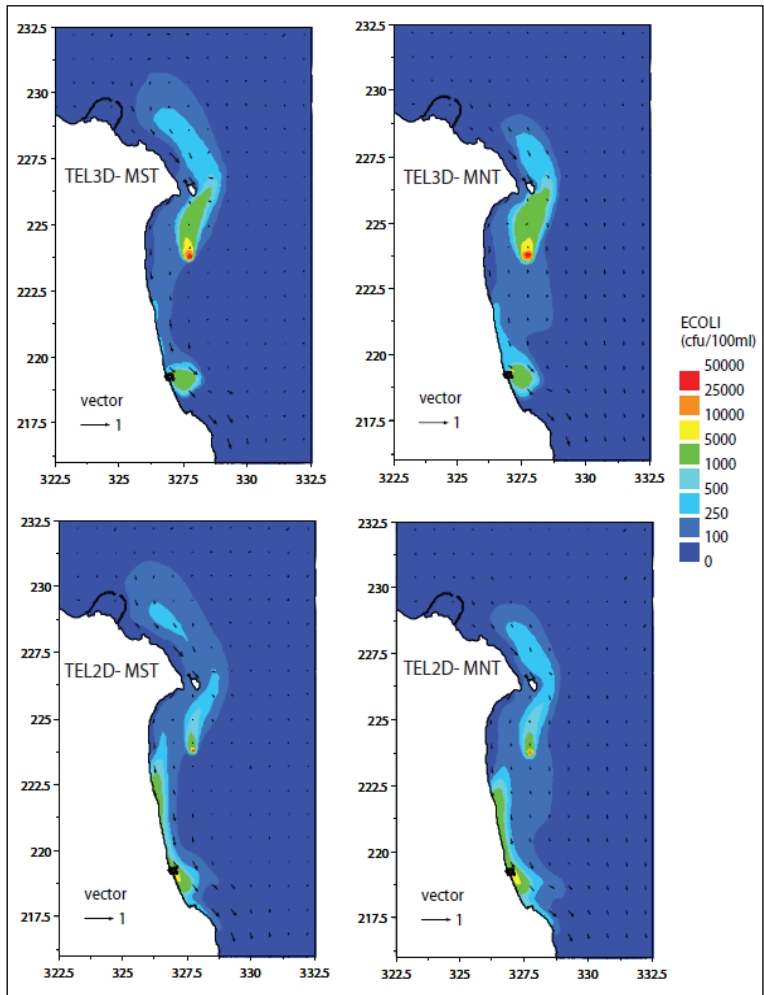


Figure5. Wet weather conditions: E. coli distributions of TELEMAC-2D and TELEMAC-3D

5. Conclusions

This study applies and compares two hydrodynamic and water quality models; a depth-averaged (TELEMAC-2D) and a three-dimensional model (TELEMAC-3D) on their performance in simulating the transport and fate of *Escherichia coli* (a main microbial bathing water quality indicator) in the coastal waters of Bray subjected to sewage discharges and catchment runoff. The models first calibrated and validated against hydrodynamic and water quality data, were then used to simulate *Escherichia coli* distribution patterns based on mean spring and mean neap tides for dry and wet weather scenarios. The hydrodynamic calibration yielded a good match for both TELEMAC-2D and TELEMAC-3D with measured velocities and *E. coli*, however TELEMAC-2D resulted in a lower value for decay rate (higher T90 value) than TELEMAC-3D in order to match the provide a match to measured *E. coli* concentrations. *E. coli* surface distributions at the time of HW resulted in TELEMAC-2D plumes that were lesser in extent and concentrations than those of TELEMAC-3D. This is perhaps because depth-averaged hydrodynamics tend to underestimate the surface water velocity resulting in lower concentrations of *E. coli* at the water surface compared to TELEMAC-3D. The wet weather scenarios of both TELEMAC-2D and TELEMAC-3D exhibited high *E. coli* concentrations at the water surface that exceed the Sufficient limit of the bathing water directive, the latter finding highlights the need for including Ultra Violet disinfection in the treatment process at Shanganagh STW.

Acknowledgements

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