Comparison of open and closed loop process identification techniques in the time domain

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COMPARISON OF OPEN- AND CLOSED-LOOP PROCESS IDENTIFICATION TECHNIQUES IN THE TIME DOMAIN.

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Abstract: This paper describes seven methods to identify a mathematical model for a real process with a time delay. The process is the Process Trainer, PT326 from Feedback Instruments Limited. Six of the methods use the step response data and one of the methods uses the impulse response data for identification.

Introduction: The dynamics of a process can be determined from the response of the process to pulses, steps, ramps, or other deterministic signals. The dynamics of a linear system are, in principle, uniquely given from such a transient response experiment. Such experiments require that the system be at rest before the input is applied. Models obtained from a transient experiment are sufficient for PID controller tuning.

The methods are implemented using the following tools:
- MatLab Version 5.3
- Simulink, Simulink Version 3 Library
- Humusoft Real Time Toolbox Version 3.0
- AD512 Data Acquisition Card plugged into ISA port
- Process Trainer PT326
- 37-pin D-type connector, 37-way cable and connector block

Keywords: Real-time Identification; First-order-plus-dead-time model; Second-order-plus-dead-time model.

Open Loop Methods: The first three methods, of the seven investigated, use the open loop step response data to identify a model.

These methods are:
1. Deduction of model directly from process response (graphical approach)
2. Two-point algorithm
3. Area method

A step is applied to the process and the resulting data from the process is examined to deduce the required information. The model obtained is a parametric model, the first-order-plus-dead-time (FOPDT) model. This model is characterised by three parameters: the static gain $K_m$, the time constant $\tau_m$, and the dead time $d_m$. The model is by far the most commonly used model for PID controller tuning. The process model transfer function is shown in equation 1.

$$G_m(s) = \frac{K_m e^{-d_m s}}{1 + \tau_m s}$$  \hspace{1cm} (1)

In the graphical approach, the process gain is determined by dividing the steady state output by the input set-point value. The time constant is the time taken for the output to reach 63% of the final value and the dead time is the time interval between the input being applied to the system and the output responding to this signal.

In the two-point algorithm approach, the steady state gain is determined as in the graphical method. The time taken for the process output to reach 28% and 63% of the final steady state output is used to determine the time constant and the dead time. The “Two point” algorithm is based on simultaneous equations 2 and 3. $T_D$ is the dead time and $T_C$ is the time constant.

$$T_{63} = T_D + T_C$$  \hspace{1cm} (2)

$$T_{28} = T_D + \frac{T_C}{3}$$  \hspace{1cm} (3)

Figure 1 shows the MatLab/Simulink/Humusoft file used in the open-loop system identification tests.
The third method is the area method and is based on integrals of the step response. The algorithm integrates areas from the open loop step response data and from the resulting values, the time constant and the dead time are calculated. The average residence time, $T_{ar}$, is the sum of the dead time and the time constant.

Results of the estimated parameter values:

**Graphically**  \( K_m = 1.1487, \quad \tau_m = 0.6 \text{ sec.}, \quad d_m = 0.26 \text{ sec.} \)

**Two-Point Algorithm**  \( K_m = 1.1459, \quad \tau_m = 0.525 \text{ sec.}, \quad d_m = 0.355 \text{ sec.} \)

**Area Method**  \( K_m = 1.1329, \quad \tau_m = 0.3568 \text{ sec.}, \quad d_m = 0.4009 \text{ sec.} \)

The fourth identification technique uses the Method of Moments algorithms to identify the three parameters for equation 1. A unit impulse is applied to the process (in open loop) and algorithms determine the parameters from the impulse response data. The area under the impulse response curve determines the process gain. This area value is also used to determine the time constant. When the time constant is known, the dead time is found by subtracting the time constant from the average residence time. There are two different estimates here as two different pulse dimensions are used.

Results of the estimated parameter values:

**Method of Moments (1)**  \( K_m = 1.1488, \quad \tau_m = 0.6867 \text{ sec.}, \quad d_m = 1.0713 \text{ sec.} \)

**Method of Moments (2)**  \( K_m = 1.3143, \quad \tau_m = 0.9374 \text{ sec.}, \quad d_m = 0.5583 \text{ sec.} \)

**Closed Loop Methods:** The next three methods implemented on the process trainer are closed-loop methods. The MatLab/Simulink/Humusoft file in figure 3 is used.

The first closed loop identification technique is based on a paper by Bogere and Ozgen [1] and identifies a four-parameter model shown in equation 4. The test is carried out in closed-loop under proportional control.
This is a second order plus dead time model. $K_m$ is the process model gain, $d_m$ is the process model dead time and the two time constants are denoted by $\tau_1$ and $\tau_2$. The proportional gain is set so that the process output has an oscillatory response as shown in figure 4.

$$G_m(s) = \frac{K_m e^{-d_m s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$  \hspace{1cm} \text{(4)}$$

Figure 4. Under-damped transient response, for a set point input of magnitude $A$ [1].

The time delay, $d_m$, is taken directly as the time interval between the time when the set-point input is made to the process and the time when the output from the process begins to respond to the input. A modified three-term Taylor approximation of the exponential delay term in the closed loop characteristic equation is used. This allows a second order closed loop approximation to be written in terms of $K_m$, $d_m$, $\tau$, and $\zeta$. The parameters $\tau$ and $\zeta$ can be expressed in terms of the measurable quantities $\Delta t$ and $Y_m, Y_{pl}, Y_{pl}, Y_{m0}$ on the response curve. Hence, the model parameters, $K_m$, $\tau_1$ and $\tau_2$ are estimated as

$$K_m = \frac{|Y_m - Y_0|}{K_r (A - |Y_m - Y_0|)}$$  \hspace{1cm} \text{(5)}$$

$$\tau_1 = \frac{1}{\pi} \sqrt{1 - \zeta^2} (1 + K) - 0.5 a K d_m, \beta = \left(\beta_1 + \beta_2 + \beta_3\right)^{\frac{1}{3}}$$  \hspace{1cm} \text{(6)}$$

$$\tau_2 = \alpha - \beta$$  \hspace{1cm} \text{(7)}$$

where

$$\alpha = \frac{\Delta t}{\pi} \sqrt{1 - \zeta^2} (1 + K) - 0.5 a K d_m.$$  \hspace{1cm} \text{(8)}$$

The parameters in equation 8 are defined in the 1989 paper by Bogere & Ozgen [1].

Results of the estimated parameter values:
Bogere & Ozgen [1] \hspace{1cm} $K_m = 0.857$, $\tau_1 = 0.7018$ secs., $\tau_2 = 0.2232$ secs., $d_m = 0.25$ sec.

Alternatively, a method described by Mamat and Fleming [2] is used to identify a first order plus dead time model in closed-loop under PI control. The model structure is shown in equation 1. If the PI controller parameters $K_C$ and $T_I$ are chosen such that the closed-loop response exhibits an under-damped response, as shown in Figure 5, then by using the Pade approximation for the dead-time term, $e^{-d_m s}$, in the denominator, the closed-loop response can be approximated by a second order plus dead-time transfer function:

$$G_{CL}(s) = \frac{C(s)}{R(s)} = \frac{K e^{-d_m s}}{\tau^2 s^2 + 2\zeta \tau s + 1}$$  \hspace{1cm} \text{(9)}$$

From the closed loop step response data, five characteristic points are used to determine the second order plus dead-time model, equation 9, and subsequently, the frequency response of the closed-loop system. Knowing the dynamics of the closed-loop system and the dynamics of the controller, the open-loop dynamics of the process can be determined by separating the dynamics of the controller from the closed-loop dynamics. The equations to determine $K, d, \tau$ and $\zeta$ for equation 9 are shown in the appendix.

Results of the estimated parameter values:
Mamat & Fleming [2] \hspace{1cm} $K_m = 1.0564$, $\tau_m = 0.4461$ secs., $d_m = 0.4991$. 
The third closed loop identification method implemented on the process trainer is that proposed by Suganda, Krishnaswamy and Rangaiah [3] to identify a second order plus dead time process model. The system is in closed-loop under PI control. In this experiment, the same five characteristic points, as shown in figure 5, that are used in the Mamat & Fleming [2] technique are taken to determine the second-order-plus-dead-time model of the overall closed loop system. The phase crossover frequency and the magnitude at this frequency are then determined; the four parameters for the second-order-plus-dead-time process model are subsequently calculated (see appendix).

Results of the estimated parameter values:

Suganda et al. [3] \[ K_m = 0.9996, \tau_m = 0.258, \tau_m = 1.0697, d_m = 0.2759 \text{ sec.} \]

**Validation:** The results of the parameter estimation for each of the identification techniques mentioned previously are validated in the time domain and the frequency domain. The most accurate open loop and closed loop identification methods are demonstrated in this report. These are the Two-point and the Suganda et al. methods.

Figure 5. Typical under-damped closed-loop step response under PI control.

Figure 6. Open loop step response of Process Trainer, PT326, and models.
Figure 7. Closed loop step response of Process Trainer, PT326, and models.

Figure 8. Comparison of Nyquist plots for PT326 and two “best-fit” models.
Conclusion: The results of the seven experiments to identify a process model are compared. It is concluded that the “best-fit” between model and process is achieved by using the Two-point method or the Suganda et al. [3] method. The time-domain and frequency-domain comparisons of model and process demonstrate the accuracy of the models in figures 6, 7, 8 and 9. The Two-point method identifies a first-order-lag-plus-dead-time model and is a relatively straightforward method carried out in open loop. A disadvantage of open loop identification is that the process has to be taken out of commission while the test is being carried out. The Suganda et al. [3] method is a closed loop test carried out while the loop is under PI control. The test identifies a second-order-plus-dead-time process model. It contains more complicated algorithms than the most of the other tests mentioned. However, since most feedback loops in practise involve PI controllers, an added advantage of this method is that the test data for retuning could be obtained during normal operation, for example, while switching from one operating level to another.

References

Appendix
Mamat & Fleming time domain solutions for equation 9. Note that A is the magnitude of the set-point change.

\[
K = \frac{C_{ss}}{A}; \quad \rho = -\frac{1}{2\pi} \ln \left(\frac{C_{p1} - C_{ss}}{C_{p1} - C_{ss}}\right); \quad \zeta = \frac{\rho^2}{1 + \rho^2}; \quad d = \frac{S_c}{C_{ss}} - 2\zeta \tau; \quad S_c = \int_0^\infty [C_{ss} - C(t)] dt
\]

The equations to determine the FOLPD parameters \(K_m\), \(\zeta_m\) and \(d_m\) are given in the 1995 Mamat & Fleming [2] paper.

Suganda, Krishnaswamy and Rangaiah method of evaluating \(\zeta_m\). The equations to determine \(K_m\), \(\tau_m\) and \(d_m\) are given in the Suganda et al. 1998 paper [3].

\[
\zeta_m = \frac{1}{2\tau_m} \times \left(\frac{(KC_k m)^2 (\omega_c^2 T_i^2 + 1)(M + 1)^2}{M^2 \omega_c^2 T_i^2} - \tau_m \omega_c^2 + 2\tau_m^2\right)
\]