1999-01-01

PI and PID controller tuning rules for time delay processes: a summary. Part 2: PID controller tuning rules

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Abstract

A summary of tuning rules for the PID control of single input, single output (SISO) processes with time delay, modeled in stable first order lag plus time delay (FOLPD) form, is provided in this part of the paper.

Keywords: PID, tuning rules, time delay.

1. Introduction

This part of the paper summarises the most directly applicable tuning rules for PID controllers that have been developed to compensate stable SISO processes with time delay, modeled in FOLPD form. The structure of this part of the paper is similar to that of Part 1.

The ideal continuous time domain PID controller for a SISO process is expressed in the Laplace domain as follows:

$$U(s) = G_c(s)E(s) \tag{1}$$

with

$$G_c(s) = K_c(1 + \frac{1}{T_i s} + T_d s) \tag{2}$$

and with K_c = proportional gain, T_i = integral time constant and T_d = derivative time constant. A number of other PID controller structures may be described. One typical example of such a structure is the classical form of the PID controller:

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + T_d s}{1 + \frac{T_d s}{N}}\right) \tag{3}$$

Tuning rules for these and other such PID controller structures are explicitly indicated in the table. A list of symbols used in the paper is provided in Appendix 1.

2. PID tuning rules - FOLPD model

Rule	K_c	T_i	T_d	Comment
Ideal controller	$G_c(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right)$			
Process reaction				
Ziegler and Nichols [1]	$[\frac{1.2T_m}{K_m \tau_m}, \frac{2T_m}{K_m \tau_m}]$	$2\tau_m$	$0.5\tau_m$	Quarter decay ratio
Astrom and Hagglund [2]	$\frac{0.94T_m}{K_m \tau_m}$	$2\tau_m$	$0.5\tau_m$	ultimate cycle Ziegler-Nichols equivalent

Parr [3]	$\frac{1.25T_m}{K_m\tau_m}$	$2.5\tau_m$	$0.4\tau_m$	
Rule	K_c	T_i	T_d	Comment
Chien *et al.* [4] - regulator	$\frac{0.95T_m}{K_m\tau_m}$	$2.38\tau_m$	$0.42t_m$	0% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Chien *et al.* [4] - regulator	$\frac{1.2T_m}{K_m\tau_m}$	$2\tau_m$	$0.42\tau_m$	20% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Chien *et al.* [4] - servo	$\frac{0.6T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	0% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Chien *et al.* [4] - servo	$\frac{0.95T_m}{K_m\tau_m}$	$1.36T_m$	$0.47\tau_m$	20% overshoot; $0.11 < \frac{\tau_m}{T_m} < 1$
Three constraints method - Murrill [5] $0.1 \le \frac{\tau_m}{T_m} \le 1$	$\frac{1.370}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.950}$	$\frac{T_m}{1.351}\left(\frac{\tau_m}{T_m}\right)^{0.738}$	$0.365T_m\left(\frac{\tau_m}{T_m}\right)^{0.950}$	Quarter decay ratio; minimum integral error (servo mode); $\frac{K_c K_m T_d}{T_m} = 0.5$
Cohen and Coon [6]	$\frac{1}{K_m}\left(1.35\frac{T_m}{\tau_m} + 0.25\right)$	$T_m\left(\frac{2.5\frac{\tau_m}{T_m} + 0.46\left(\frac{\tau_m}{T_m}\right)^2}{1 + 0.61\frac{\tau_m}{T_m}}\right)$	$\frac{0.37\tau_m}{1 + 0.2\frac{\tau_m}{T_m}}$	Quarter decay ratio
Sain and Ozgen [7]	$\frac{1}{K_m}\left(0.6939\frac{T_m}{\tau_m} + 0.1814\right)$	$\frac{0.8647T_m + 0.226\tau_m}{\frac{T_m}{\tau_m} + 0.8647}$	$\frac{0.0565T_m}{0.8647\frac{T_m}{\tau_m} + 0.226}$	
Regulator				
Minimum IAE - Murrill [5]	$\frac{1.435}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.878}\left(\frac{T_m}{\tau_m}\right)^{0.749}$	$0.482T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}$	$0.1 < \frac{\tau_m}{T_m} \le 1$
Modified minimum IAE - Cheng and Hung [8]	$\frac{3}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.921}$	$\frac{T_m}{0.878}\left(\frac{T_m}{\tau_m}\right)^{0.749}$	$0.482T_m\left(\frac{\tau_m}{T_m}\right)^{1.137}$	
Minimum ISE - Murrill [5]	$\frac{1.495}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.945}$	$\frac{T_m}{1.101}\left(\frac{T_m}{\tau_m}\right)^{0.771}$	$0.56T_m\left(\frac{\tau_m}{T_m}\right)^{1.006}$	$0.1 < \frac{\tau_m}{T_m} \le 1$
Minimum ISE - Zhuang and Atherton [9]	$\frac{1.473}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.970}$	$\frac{T_m}{1.115}\left(\frac{T_m}{\tau_m}\right)^{0.753}$	$0.55T_m\left(\frac{\tau_m}{T_m}\right)^{0.948}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.524}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.735}$	$\frac{T_m}{1.130}\left(\frac{T_m}{\tau_m}\right)^{0.641}$	$0.552T_m\left(\frac{\tau_m}{T_m}\right)^{0.851}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ITAE - Murrill [5]	$\frac{1.357}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.947}$	$\frac{T_m}{0.842}\left(\frac{T_m}{\tau_m}\right)^{0.738}$	$0.381T_m\left(\frac{\tau_m}{T_m}\right)^{0.995}$	$0.1 < \frac{\tau_m}{T_m} \le 1$
Minimum ISTSE - Zhuang and Atherton [9]	$\frac{1.468}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.970}$	$\frac{T_m}{0.942}\left(\frac{T_m}{\tau_m}\right)^{0.725}$	$0.443T_m\left(\frac{\tau_m}{T_m}\right)^{0.939}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.515}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.730}$	$\frac{T_m}{0.957}\left(\frac{T_m}{\tau_m}\right)^{0.598}$	$0.444T_m\left(\frac{\tau_m}{T_m}\right)^{0.847}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$

Minimum ISTES - Zhuang and Atherton [9]	$\frac{1.531}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.960}$	$\frac{T_m}{0.971}\left(\frac{T_m}{\tau_m}\right)^{0.746}$	$0.413T_m\left(\frac{\tau_m}{T_m}\right)^{0.933}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$

Rule	K_c	T_i	T_d	Comment
Minimum ISTES - Zhuang and Atherton [9] (continued)	$\frac{1.592}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.705}$	$\frac{T_m}{0.957}\left(\frac{T_m}{\tau_m}\right)^{0.597}$	$0.414T_m\left(\frac{\tau_m}{T_m}\right)^{0.850}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Servo				
Minimum IAE - Rovira *et al*. [10]	$\frac{1.086}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.869}$	$\frac{T_m}{0.740 - 0.13\frac{\tau_m}{T_m}}$	$0.348T_m\left(\frac{\tau_m}{T_m}\right)^{0.914}$	$0.1 < \frac{\tau_m}{T_m} \le 1$
Minimum IAE - Wang *et al*. [11]	$\frac{\left(0.7645 + \frac{0.6032}{\tau_m/T_m}\right)\left(T_m + 0.5\tau_m\right)}{K_m\left(T_m + \tau_m\right)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISE - Wang *et al*. [11]	$\frac{\left(0.9155 + \frac{0.7524}{\tau_m/T_m}\right)\left(T_m + 0.5\tau_m\right)}{K_m\left(T_m + \tau_m\right)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISE - Zhuang and Atherton [9]	$\frac{1.048}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.897}$	$\frac{T_m}{1.195 - 0.368\frac{\tau_m}{T_m}}$	$0.489T_m\left(\frac{\tau_m}{T_m}\right)^{0.888}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.154}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.567}$	$\frac{T_m}{1.047 - 0.220\frac{\tau_m}{T_m}}$	$0.490T_m\left(\frac{\tau_m}{T_m}\right)^{0.708}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ITAE - Rovira *et al*. [10]	$\frac{0.965}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.85}$	$\frac{T_m}{0.796 - 0.1465\frac{\tau_m}{T_m}}$	$0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}$	$0.1 \le \frac{\tau_m}{T_m} \le 1$
Modified minimum ITAE - Cheng and Hung [8]	$\frac{1.2}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.855}$	$\frac{T_m}{0.796 - 0.147\frac{\tau_m}{T_m}}$	$0.308T_m\left(\frac{\tau_m}{T_m}\right)^{0.929}$	Damping factor of closed loop system = 0.707.
Minimum ITAE - Wang *et al*. [11]	$\frac{\left(0.7303 + \frac{0.5307}{\tau_m/T_m}\right)\left(T_m + 0.5\tau_m\right)}{K_m\left(T_m + \tau_m\right)}$	$T_m + 0.5\tau_m$	$\frac{0.5T_m\tau_m}{T_m + 0.5\tau_m}$	$0.05 < \frac{\tau_m}{T_m} < 6$
Minimum ISTSE - Zhuang and Atherton [9]	$\frac{1.042}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.897}$	$\frac{T_m}{0.987 - 0.238\frac{\tau_m}{T_m}}$	$0.385T_m\left(\frac{\tau_m}{T_m}\right)^{0.906}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.142}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.579}$	$\frac{T_m}{0.919 - 0.172\frac{\tau_m}{T_m}}$	$0.384T_m\left(\frac{\tau_m}{T_m}\right)^{0.839}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Minimum ISTES - Zhuang and Atherton [9]	$\frac{0.968}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.904}$	$\frac{T_m}{0.977 - 0.253\frac{\tau_m}{T_m}}$	$0.316T_m\left(\frac{\tau_m}{T_m}\right)^{0.892}$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
	$\frac{1.061}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.583}$	$\frac{T_m}{0.892 - 0.165\frac{\tau_m}{T_m}}$	$0.315T_m\left(\frac{\tau_m}{T_m}\right)^{0.832}$	$1.1 \le \frac{\tau_m}{T_m} \le 2.0$
Direct synthesis				
Regulator - min. IAE - Smith and Corripio [12]	$\frac{T_m}{K_m\tau_m}$	T_m	$0.5\tau_m$	$0.1 \le \frac{\tau_m}{T_m} \le 1.5$

Servo - min. IAE - Smith and Corripio [12]	$\frac{5T_m}{6K_m\tau_m}$	T_m	$0.5\tau_m$	$0.1 \le \frac{\tau_m}{T_m} \le 1.5$
Servo - 5% overshoot - Smith and Corripio [12]	$\frac{T_m}{2K_m\tau_m}$	T_m	$0.5\tau_m$	

Rule	K_c	T_i	T_d	Comment
Suyama [13]	$\frac{1}{K_m}\left[0.7236\frac{T_m}{\tau_m}+0.2236\right]$	$T_m + 0.309\tau_m$	$\frac{2.236T_m\tau_m}{7.236T_m + 2.236\tau_m}$	
Gain and phase margin - Zhuang and Atherton [9]	$mK_u\cos(\phi_m)$, $m = 0.614(1-0.233e^{-0.347K_mK_u})$ $\phi_m = 33.8^0\left(1-0.97e^{-0.45K_mK_u}\right)$	$\alpha\frac{\tan(\phi_m)+\sqrt{\frac{4}{\alpha}+\tan^2(\phi_m)}}{2\omega_u}$, $\alpha = 0.413(3.302K_mK_u+1)$	$\frac{\tan(\phi_m)+\sqrt{\frac{4}{\alpha}+\tan^2(\phi_m)}}{2\omega_u}$	Gain margin = 2, phase margin = 60^0 $0.1 \le \frac{\tau_m}{T_m} \le 2.0$
Abbas [14]	$\frac{0.177+0.348\left(\frac{\tau_m}{T_m}\right)^{-1.002}}{0.531-0.359V^{0.713}}$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m+\tau_m}$	V=fractional overshoot $0 \le V \le 0.2$ $0.1 \le \frac{\tau_m}{T_m} \le 5.0$
Servo - min. ISE - Ho *et al*. [15]	$\frac{1.8578}{K_m}\frac{\phi_m^{0.0821}}{A_m^{0.9087}}\left(\frac{\tau_m}{T_m}\right)^{-0.9471}$	$T_i^{(1)}$ [1]	$\frac{0.4899T_m\phi_m^{0.1457}}{A_m^{0.0845}}\left(\frac{\tau_m}{T_m}\right)^{1.0264}$	$A_m \in [2,5]$, $\phi_m \in [30^0, 60^0]$, $0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Regulator - min. ISE - Ho *et al*. [15]	$\frac{1.0722}{K_m}\frac{\phi_m^{-0.116}}{A_m^{0.8432}}\left(\frac{\tau_m}{T_m}\right)^{-0.908}$	$\frac{1.2497T_m\phi_m^{1.0082}}{A_m^{0.2099}}\left(\frac{\tau_m}{T_m}\right)^{0.3678}$	$\frac{0.4763T_m\phi_m^{-0.328}}{A_m^{0.0961}}\left(\frac{\tau_m}{T_m}\right)^{1.0317}$	$A_m \in [2,5]$, $\phi_m \in [30^0, 60^0]$, $0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Robust				
Brambilla *et al*. [16]	$\frac{1}{K_m}\left(\frac{T_m+0.5\tau_m}{\lambda\tau_m}\right)$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m+\tau_m}$	$0.1 \le \frac{\tau_m}{T_m} \le 10$; $l \approx 0.35$
Rivera *et al*. [17]	$\frac{1}{K_m}\left(\frac{T_m+0.5\tau_m}{\lambda+0.5\tau_m}\right)$	$T_m + 0.5\tau_m$	$\frac{T_m\tau_m}{2T_m+\tau_m}$	$\lambda > 0.1T_m$, $\lambda \ge 0.8\tau_m$.
Fruehauf *et al*. [18]	$\frac{5T_m}{9\tau_mK_m}$	$5\tau_m$	$\le 0.5\tau_m$	$\frac{\tau_m}{T_m} < 0.33$
	$\frac{T_m}{2\tau_mK_m}$	T_m	$\le 0.5\tau_m$	$\frac{\tau_m}{T_m} \ge 0.33$
Lee *et al*. [19]	$\frac{T_i}{K_m(\lambda+\tau_m)}$	$T_m + \frac{\tau_m^2}{2(\lambda+\tau_m)}$	$\frac{\tau_m^2}{2(\lambda+\tau_m)}\left(1-\frac{\tau_m}{3T_i}\right)$	$\lambda = \frac{\tau_m}{3}$
Ultimate cycle				
Regulator - minimum IAE - Pessen [20]	$0.7K_u$	$0.4T_u$	$0.149T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 1.0$
Servo - minimum ISTSE - Zhuang and	$0.509K_u$	$0.051(3.302K_mK_u+1)T_u$	$0.125T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$

[1] $T_i^{(1)} = \frac{0.0211T_m\left[1+0.3289A_m+6.4572\phi_m+25.1914(\tau_m/T_m)\right]}{1+0.0625A_m-0.8079\phi_m+0.347(\tau_m/T_m)}$

Atherton [9]				
Regulator - minimum ISTSE - Zhuang and Atherton [9]	$\frac{4.434K_mK_u - 0.966}{5.12K_mK_u + 1.734}K_u$	$\frac{1.751K_mK_u - 0.612}{3.776K_mK_u + 1.388}T_u$	$0.144T_u$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$

Rule	K_c	T_i	T_d	Comment
Servo - nearly minimum IAE, ITAE - Hwang and Fang [21]	$\left[c_1 + c_2\frac{\tau_m}{T_m} + c_3\left(\frac{\tau_m}{T_m}\right)^2\right]K_u$ $c_1 = 0.537, c_2 = -0.0165$ $c_3 = 0.00173$	$\frac{K_c}{K_u\omega_u\left[c_4 + c_5\frac{\tau_m}{T_m} + c_6\left(\frac{\tau_m}{T_m}\right)^2\right]}$ $c_4 = 0.0503, c_5 = 0.163$ $c_6 = -0.0389$	$\left[c_7 + c_8\frac{\tau_m}{T_m} + c_9\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c}$ $c_7 = 0.350, c_8 = -0.0344$ $c_9 = 0.00644$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$ - decay ratio = 0.03
Regulator - nearly minimum IAE, ITAE - Hwang and Fang [21]	$\left[c_1 + c_2\frac{\tau_m}{T_m} + c_3\left(\frac{\tau_m}{T_m}\right)^2\right]K_u$ $c_1 = 0.802, c_2 = -0.154$ $c_3 = 0.0460$	$\frac{K_c}{K_u\omega_u\left[c_4 + c_5\frac{\tau_m}{T_m} + c_6\left(\frac{\tau_m}{T_m}\right)^2\right]}$ $c_4 = 0.190, c_5 = 0.0532$ $c_6 = -0.00509$	$\left[c_7 + c_8\frac{\tau_m}{T_m} + c_9\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c}$ $c_7 = 0.421, c_8 = 0.00915$ $c_9 = -0.00152$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$ - decay ratio = 0.12
Simultaneous Servo/regulator - nearly minimum IAE, ITAE - Hwang and Fang [21]	$\left[c_1 + c_2\frac{\tau_m}{T_m} + c_3\left(\frac{\tau_m}{T_m}\right)^2\right]K_u$ $c_1 = 0.713, c_2 = -0.176$ $c_3 = 0.0513$	$\frac{K_c}{K_u\omega_u\left[c_4 + c_5\frac{\tau_m}{T_m} + c_6\left(\frac{\tau_m}{T_m}\right)^2\right]}$ $c_4 = 0.149, c_5 = 0.0556$ $c_6 = -0.00566$	$\left[c_7 + c_8\frac{\tau_m}{T_m} + c_9\left(\frac{\tau_m}{T_m}\right)^2\right]\frac{K_u}{\omega_u K_c}$ $c_7 = 0.371, c_8 = -0.0274$ $c_9 = 0.00557$	$0.1 \le \frac{\tau_m}{T_m} \le 2.0$
McMillan [22]	$\frac{1.415}{K_m}\frac{T_m}{\tau_m}\left\{\frac{1}{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}}\right\}$	$\tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\}$	$0.25\tau_m\left\{1+\left(\frac{T_m}{T_m+\tau_m}\right)^{0.65}\right\}$	Tuning rules developed from K_u, T_u
Tan *et al*. [23]	$\frac{K_u}{A_m}\cos\phi_m$	αT_d	$T_u\frac{\tan\phi_m + \sqrt{\frac{4}{\alpha} + \tan^2\phi_m}}{4\pi}$	$A_m = 2$, $\phi_m = 45^0$; α chosen arbitrarily
	$\frac{K_\phi}{A_m}$	$\frac{rK_\phi\left(\omega_u{}^2 - \omega_\phi{}^2\right)}{\omega_u\omega_\phi{}^2\sqrt{K_u{}^2 - r^2K_\phi{}^2}}$	$\frac{1}{\omega_\phi{}^2T_i}$	Arbitrary A_m, ϕ_m at ω_ϕ; $r = 0.1 + 0.9(K_u/K_\phi)$
<u>Ideal controller with set-point weighting</u>	$G_c(s) = K_c\left(b + \frac{1}{T_is} + T_ds\right)$			
Maximum sensitivity				
Astrom and Hagglund [2] - maximum sensitivity = 1.4	$\frac{3.8e^{-8.4\tau+7.3\tau^2}T_m}{K_m\tau_m}$, $\tau = \tau_m/(\tau_m + T_m)$	$5.2\tau_m e^{-2.5\tau-1.4\tau^2}$ or $0.46T_m e^{2.8\tau-2.1\tau^2}$	$0.89\tau_m e^{-0.37\tau-4.1\tau^2}$ or $0.077T_m e^{5.0\tau-4.8\tau^2}$	$b = 0.40e^{0.18\tau+2.8\tau^2}$ $0.14 \le \frac{\tau_m}{T_m} \le 5.5$
Astrom and Hagglund [2] - maximum sensitivity = 2.0	$\frac{8.4e^{-9.6\tau+9.8\tau^2}T_m}{K_m\tau_m}$	$3.2\tau_m e^{-1.5\tau-0.93\tau^2}$ or $0.28T_m e^{3.8\tau-1.6\tau^2}$	$0.86\tau_m e^{-1.9\tau-0.44\tau^2}$ or $0.076T_m e^{3.4\tau-1.1\tau^2}$	$b = 0.22e^{0.65\tau+0.051\tau^2}$ $0.14 \le \frac{\tau_m}{T_m} \le 5.5$
<u>Classical controller</u>	$G_c(s) = K_c\left(1 + \frac{1}{T_is}\right)\left(\frac{1 + T_ds}{1 + \frac{T_ds}{N}}\right)$			
Process reaction				
Hang *et al*. [24]	$\frac{0.83T_m}{K_m\tau_m}$	$1.5\tau_m$	$0.25\tau_m$	Foxboro EXACT controller 'pretune'; N=10

Witt and Waggoner [25]	$[\frac{0.6T_m}{K_m\tau_m}, \frac{T_m}{K_m\tau_m}]$	τ_m	τ_m	Equivalent to Ziegler and Nichols [1]; $N = [10,20]$
St. Clair [26]	$\frac{T_m}{K_m\tau_m}$	$5\tau_m$	$0.5\tau_m$	'aggressive' tuning
	$\frac{0.5T_m}{K_m\tau_m}$	$5\tau_m$	$0.5\tau_m$	'conservative' tuning

Rule	K_c	T_i	T_d	Comment
Regulator				
Minimum IAE - Kaya and Scheib [27]	$\frac{0.98089}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.76167}$	$\frac{T_m}{0.91032}\left(\frac{T_m}{\tau_m}\right)^{1.05221}$	$0.59974T_m\left(\frac{\tau_m}{T_m}\right)^{0.89819}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ISE - Kaya and Scheib [27]	$\frac{1.11907}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.89711}$	$\frac{T_m}{0.7987}\left(\frac{T_m}{\tau_m}\right)^{0.9548}$	$0.54766T_m\left(\frac{\tau_m}{T_m}\right)^{0.87798}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ITAE - Kaya and Scheib [27]	$\frac{0.77902}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.06401}$	$\frac{T_m}{1.14311}\left(\frac{T_m}{\tau_m}\right)^{0.70949}$	$0.57137T_m\left(\frac{\tau_m}{T_m}\right)^{1.03826}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Servo				
Minimum IAE - Kaya and Scheib [27]	$\frac{0.65}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.04432}$	$\frac{T_m}{0.9895 + 0.09539\frac{\tau_m}{T_m}}$	$0.50814T_m\left(\frac{\tau_m}{T_m}\right)^{1.08433}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ISE - Kaya and Scheib [27]	$\frac{0.71959}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.03092}$	$\frac{T_m}{1.12666 - 0.18145\frac{\tau_m}{T_m}}$	$0.54568T_m\left(\frac{\tau_m}{T_m}\right)^{0.86411}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ITAE - Kaya and Scheib [27]	$\frac{1.12762}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.80368}$	$\frac{T_m}{0.99783 + 0.02860\frac{\tau_m}{T_m}}$	$0.42844T_m\left(\frac{\tau_m}{T_m}\right)^{1.0081}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Direct synthesis				
Tsang and Rad [28]	$\frac{0.809T_m}{K_m\tau_m}$	T_m	0.	N=5
Tsang . [29]	$\frac{\ }{K_m\tau_m}$	$_m$	$0\ 25\tau_m$	N = 2.5

	a	ξ		ξ	a			ξ	a			ξ	
	1.682	0.0	1.161	0.2	0.859	0.4	0.669	0.6	0.543	0.8	0.457	1.0	
	1.383	0.1	0.992	0.3	0.754	0.5	0.600	0.7	0.496	0.9			

Robust				
Chien [30]	$\frac{1}{K_m}\left(\frac{T_m}{\lambda + 0.5\tau_m}\right)$	T_m	$0.5\tau_m$	$\lambda = [\tau_m, T_m]$; N=10
Chien [30]	$\frac{1}{K_m}\left(\frac{0.5\tau_m}{\lambda + 0.5\tau_m}\right)$	$0.5\tau_m$	T_m	$\lambda = [\tau_m, T_m]$; N=10
Ultimate cycle				
Minimum IAE regulator - Shinskey [31]	$\frac{K_m\tau_m}{0.3\tau_m - 0.32T_u}$	$T_u\left(0.15\frac{T_u}{\tau_m} - 0.05\right)$	$0.14T_u$	
Minimum IAE regulator - Shinskey [32]	$0.95T_m/K_m\tau_m$ $0.95T_m/K_m\tau_m$ $1.14T_m/K_m\tau_m$ $1.39T_m/K_m\tau_m$	$1.43\tau_m$ $1.17\tau_m$ $1.03\tau_m$ $0.77\tau_m$	$0.52\tau_m$ $0.48\tau_m$ $0.40\tau_m$ $0.35\tau_m$	$\tau_m/T_m = 0.2$ $\tau_m/T_m = 0.5$ $\tau_m/T_m = 1$ $\tau_m/T_m = 2$

Industrial controller	$U(s) = K_c\left(1+\frac{1}{T_i s}\right)\left(R(s) - \frac{1+T_d s}{1+\frac{T_d s}{N}}Y(s)\right)$			
Regulator				
Minimum IAE - Kaya and Scheib [27]	$\frac{0.91}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.7938}$	$\frac{T_m}{1.01495}\left(\frac{T_m}{\tau_m}\right)^{1.00403}$	$0.5414 T_m\left(\frac{\tau_m}{T_m}\right)^{0.7848}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10

Rule	K_c	T_i	T_d	Comment
Minimum ISE - Kaya and Scheib [27]	$\frac{1.1147}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8992}$	$\frac{T_m}{0.9324}\left(\frac{T_m}{\tau_m}\right)^{0.8753}$	$0.56508T_m\left(\frac{\tau_m}{T_m}\right)^{0.91107}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ITAE - Kaya and Scheib [27]	$\frac{0.7058}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.8872}$	$\frac{T_m}{1.03326}\left(\frac{T_m}{\tau_m}\right)^{0.99138}$	$0.60006T_m\left(\frac{\tau_m}{T_m}\right)^{0.971}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Servo				
Minimum IAE - Kaya and Scheib [27]	$\frac{0.81699}{K_m}\left(\frac{T_m}{\tau_m}\right)^{1.004}$	$\frac{T_m}{1.09112 - 0.22387\frac{\tau_m}{T_m}}$	$0.44278T_m\left(\frac{\tau_m}{T_m}\right)^{0.97186}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ISE - Kaya and Scheib [27]	$\frac{1.1427}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.9365}$	$\frac{T_m}{0.99223 - 0.35269\frac{\tau_m}{T_m}}$	$0.35308T_m\left(\frac{\tau_m}{T_m}\right)^{0.78088}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10
Minimum ITAE - Kaya and Scheib [27]	$\frac{0.8326}{K_m}\left(\frac{T_m}{\tau_m}\right)^{0.7607}$	$\frac{T_m}{1.00268 + 0.00854\frac{\tau_m}{T_m}}$	$0.44243T_m\left(\frac{\tau_m}{T_m}\right)^{1.11499}$	$0 < \frac{\tau_m}{T_m} \le 1$; N=10

3. Conclusions

This part of the paper has presented a useable summary of the tuning rules for PID controllers that have been developed to compensate SISO processes with time delay, modeled in FOLPD form.

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Appendix 1: List of symbols used (more than once) in the paper.

A_m = gain margin
b = setpoint weighting factor
E(s) = Desired variable, R(s), minus controlled variable, Y(s)
$G_c(s)$ = PID controller transfer function
$G_p(j\omega)$ = process transfer function at frequency ω, $|G_p(j\omega)|$ = magnitude of $G_p(j\omega)$
IAE = integral of absolute error
ISE = integral of squared error
ISTES = integral of squared time multiplied by error, all to be squared
ISTSE = integral of squared time multiplied by squared error
ITAE = integral of time multiplied by absolute error
K_c = Proportional gain of the controller
K_m = Gain of the FOLPD process model
K_ϕ = Magnitude of the FOLPD process model at a phase lag of ϕ
K_u = Ultimate gain
M_s = Closed loop sensitivity
N = Indication of the amount of filtering on the derivative term
R(s) = Desired variable
T_d = Derivative time of the controller
T_i = Integral time of the controller
T_m = Time constant of the FOLPD process model
T_u = Ultimate time constant
U(s) = manipulated variable
Y(s) = controlled variable
λ = Parameter that determines robustness of compensated system.
ξ = damping factor of the compensated system
ϕ_m = phase margin
τ_m = time delay of the FOLPD process model, $\tau = \tau_m/(\tau_m + T_m)$
ω_u = ultimate (angular) frequency, ω_ϕ = angular frequency at a phase lag of ϕ