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Rethinking the Mathematics Worksheet in Higher Education: Embracing the Application of Spaced and Interleaved Practice

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Abstract

Existing research in cognitive and educational psychology warns against overlearning and promotes spaced and interleaved practice. However, the practice of blocked overlearning is still the standard design of student practice in mathematics in higher education. This is despite the fact that transitioning to spaced interleaved worksheets would require only the resources already in place for traditionally designed blocked worksheets. Consequently, changing to spaced interleaved worksheets provides a straightforward, time-effective way in which higher education mathematics educators can implement evidence-based practice. This paper is a literature review with the intention of making explicit the educational benefits of a worksheet redesign in higher education mathematics and distilling it into recommendations on how to perform such a redesign. Negative impacts of the new design are also considered and suggestions are made on how to offset these.

Keywords: Interleaved, spaced, distributed, blocked, mathematics, worksheets
Introduction

When appraising learning and retention of material in the Irish higher education system, one factor to consider is the time for which students are exposed to new material. The standard length of a semester in Ireland is 13 weeks. At a minimum, therefore, it would be expected that learned material would be retained for this duration. However, programme learning outcomes are intended to be met over the entirety of a programme and so the knowledge learned in the first year of a programme should still be retained by the end of the programme, and ideally beyond, into the further study or career of the student. With this in mind, techniques and strategies that promote long-term retention are desirable tools for today’s educators.

Students’ primary method of learning mathematics outside the classroom is in completing worksheets. In this paper, the term worksheet is used to cover such things variably named as practice exercises, tutorial sheets, problem sets or homework. Conventional mathematics worksheets tend to conform to a set format, usually comprising a large number of questions directly related to the material most recently taught. This practice, where no questions relating to previous topics are included, is known as massed practice. As is evidenced by most widely used mathematics text books (Bird, 2010; James, 2010), where more than one topic has been recently covered, massed practice is often accompanied by blocked practice—presenting a number of exercises on multiple topics grouped or “blocked” together according to topic. Some more detail is given on blocked and massed practice in the section Blocked Practice vs. Interleaved Practice. The aim of this paper is to distil some recent findings from cognitive and educational psychology that are pertinent to mathematics education to inform a redesign of conventional mathematics worksheets in higher education in order to maximise the learning effect of students’ practice. The paper is written in the form of a narrative.
Over the course of this review the authors found several promising key strategies, leading to a primary focus on distributed practice—sometimes called spaced practice—where a fixed amount of practice is spread out over a set time period rather than that same amount of practice being performed en masse in one sitting—and interleaved practice—where questions from multiple topics are shuffled rather than being blocked together by topic.

Prior knowledge (and misconceptions) with which some students embark on mathematics modules as well as students’ individual perceptions of their own learning of the current material may play a part in how students view individual learning strategies. The role of educators in encouraging students to adopt appropriate strategies, such as those mentioned above, is therefore of high importance. The notion of students’ perception of their own learning is discussed later in the paper. In the current climate of higher education in Ireland and particularly in the Institutes of Technology, lecturers are constrained by limited time and resources. In order to cater for the needs of these educators it seems essential to build up a repertoire of inexpensive and time efficient tools to help student learning without a huge resource requirement. The authors propose to show that spaced and interleaved practice is one such tool.

The remainder of this paper is organised as follows. The notions of how learning and memory retrieval occur are addressed in the next section. This is followed by a discussion on each of the strategies distributed practice, and interleaved practice. Their advantages over massed and blocked practice respectively are examined. Potential difficulties that might arise from spaced and interleaved practice are also explored, with suggestions on how students might be shielded against any such negative effects. Then follows a section on Judgements of learning.
and mathematics anxiety, where students’ perceptions of their own learning are considered. In addition, the impact of adopting spaced and interleaved practice on these perceptions as well as their effect on students’ overall attitudes towards mathematics are discussed. The final two sections present the authors’ conclusions and recommendations in terms of a new design for mathematics worksheets with the aim of incorporating best practice from the literature.

**Learning, Memory, Forgetting, and Overlearning**

It is impossible to learn without some ability to remember. In his book on learning theories, Tarpy (1997) classifies memory creation into three parts: the learning stage, the storage or retention phase, and the retrieval stage. In order to understand some of the benefits of the strategies discussed in the remainder of this paper, a brief look at current theories on memory, and perhaps more importantly forgetting, is useful. Storage theories and retrieval theories play an important role in understanding how information or memories may be forgotten. Storage theories are generally concerned with loss of memories during the storage phase. The belief is that a memory either decays over time or is interfered with by some other competing memory. Retrieval theories take a very different approach. In the eyes of the retrieval theorists the memory may survive the storage phase but be inaccessible. This failure to retrieve pertinent information manifests as forgetting. Research by Grimaldi and Karpick (2012) has shown that repeated retrieval of a piece of information enhances the subsequent retrievability of that knowledge and that even unsuccessful retrieval attempts can improve memory.

Overlearning describes the strategy of practising a skill well beyond the point of mastery. It is very often a natural consequence of massed and blocked practice, as these practices inevitably involve performing one task or technique repeatedly before moving on to the next.
Where this repetition extends beyond the point at which the student can correctly perform the task, overlearning results. Most mathematics text books and current mathematics worksheets promote the practice of overlearning through massed and blocked practice as they contain a lot of exercises of very similar problems where different topics are clearly delineated. One only needs a cursory glance at most mainstream mathematics text books to see many examples of such practice (Bird, 2010; James, 2010). Driskell, Willis and Copper (1992) presented a meta-analysis on the effects of overlearning on retention for both physical and cognitive tasks. While their results showed the beneficial effects of overlearning on short-term learning for both types of task, the authors concluded that, for cognitive tasks, the benefits of overlearning diminished over time, with no observed benefits after 38 days. To examine the effects of overlearning on mathematics, Rohrer and Taylor (2006) conducted an experiment in which 50 students (Hi Massers) were exposed to overlearning and a further 50 students (Lo Massers) were not. A total of 28 of the Hi Massers and 24 of the Lo Massers were tested after one week with the remainder of each group tested after four weeks. The authors found that overlearning produced no observable benefit to student retention of the material over either interval. In conclusion, they suggested that, in each session, students should be presented with only three to four questions of any particular type. Overlearning, although commonly employed, appears to be an inefficient use of student time if the aim is to promote long-term learning and may encourage learners to compartmentalise their learning.

**The Spacing Effect: distributing practice**

Spaced or distributed practice describes the strategy of spreading a fixed amount of practice over a set time period rather than massing that same practice into one session. For example, once students have learned a certain mathematical concept or procedure they can either practise a fixed number of relevant questions in one sitting (massed practice) or they can complete the
same number of questions spread across two or more discrete sittings (spaced practice). The Spacing Effect, first identified by Ebbinghaus in 1885 (Gilhooly, Lyddy, & Pollick, 2014), describes the finding that spaced or distributed practice results in better learning and retention of material. In 2009, Rohrer published a paper reviewing research carried out on the effects of spacing and mixed (interleaved) practice on retention. In it, the author reviews work showing spaced practice to be beneficial for test delays of one year and more (Bahrick, Bahrick, Bahrick, & Bahrick, 1993; Cepeda, Vul, Rohrer, Wixted, & Pashler, 2008). Although these experiments were not specific to mathematics, the findings are very positive indicators of the long-term retention benefits of spaced practice. In a review of the effectiveness of ten learning techniques by Dunlosky, Rawson, Marsh, Nathan and Willingham (2013), distributed practice was one of only two techniques to be rated as having high utility. The remainder were rated as having moderate or low utility.

There are three commonly held views on how the spacing effect works: (i) attention attenuation/deficient processing, (ii) study-phase retrieval and (iii) encoding variability. The deficient processing view was more recently named attention attenuation by Wahlheim, Dunlosky and Jacoby (2011) whose experimental results support the theory. It claims that when presented with many very similar examples or exercises massed together, students pay less attention, and allocate less time, to the examples appearing later in each massed block. The theory suggests that students simply find massed practice less interesting than the same content practised over spaced intervals.

The study-phase retrieval view is linked to the retrieval theories of remembering discussed in the last section. When a student works on a number of similar questions in one sitting, the techniques required for the earlier questions may still be in working memory, and so retrieval
from long-term memory is not required for subsequent questions. On the contrary, in spaced practice the earlier examples or other knowledge from the topic need to be retrieved from long-term memory in order to attempt subsequent questions. The very act of retrieving the memory strengthens it and increases the likelihood of further successful retrieval at a later time (Dunlosky et al., 2013).

The encoding variability view holds that each time a student meets the material in spaced practice it is encoded in a slightly different context and that these different contexts increase the chances of retrieval in the assessment context (Gilhooly et al. 2014). The encoding variability view is investigated and supported by Cepeda et al. (2009).

Whatever the mechanism by which the spacing effect works, there is little debate as to its efficacy. Dunlosky et al. (2013) come to the conclusion that given the strength of the spacing effect it seems reasonable to assume that multiple mechanisms may work in parallel to contribute to the effect.

The benefits of spaced practice have been shown directly in relation to the study of mathematics. Rohrer and Taylor (2006) conducted an experiment on the effects of distributed practice in mathematics in which 58 students (Spacers) were exposed to spaced practice and a further 58 students (Massers) were exposed to massed practice. A total of 29 of the Spacers and 31 of the Massers were tested after one week with the remainder of each group tested after four weeks. The results showed almost equal performance in the one-week test (with a mean result of 70% for Spacers versus 75% for Massers). However, when tested after four weeks, the Spacers significantly outperformed the Massers with mean results of 64% and 32% respectively. In addition to the superior performance by the Spacers, these results also
demonstrate a lesser decline in marks between tests for students engaging in spaced practice than those engaging in massed practice. This indicates that long-term retention of mathematics procedures is improved by spacing practice sessions. Roher and Taylor (2006) also recommend that only three or four of the assigned problems on a worksheet should relate to the material which has just been taught. More questions of the same type should then be spread across subsequent worksheets. Therefore, in any one practice session a student should complete a few problems from the current topic and the remainder of the problems should be drawn from topics covered in previous sessions.

A natural consideration in relation to spaced practice is that of timing. Several studies have focussed on investigating the optimal timings of spaced practice and post-tests (tests carried out once a set period of time has elapsed following the final practice session). Cepeda et al. (2008) studied this in great detail and derived complex relationships between optimal spacing gap or lag and retention interval. However, a certain degree of caution should be observed: an over-emphasis on optimising spacing times may make the task of designing worksheets much more complicated than necessary while possibly having a negligible effect on outcomes. In a review of nine recent studies relating to optimal spacing, Rohrer (2015) concludes that longer timeframes and several repeat exposures result in the best post-test scores once the post-test is delayed more than a month. Applying these findings to the standard thirteen-week module implies that students should be exposed to practice of each topic on more than two occasions and that the spacing between occasions lie anywhere between one and four weeks.

The practice of distributing questions on any one topic across multiple worksheets gives rise to the situation where multiple topics appear on a single worksheet. Consequently, consideration must be given as to how to arrange the questions. For instance, questions may be
organised by topic in individual blocks or may be randomly mixed or shuffled. Here too we are guided by literature from cognitive psychology.

**Blocked Practice vs. Interleaved Practice**

Arranging questions by topic is known as blocked practice. For instance, in a calculus worksheet, one might see five questions on the product rule followed by five questions on the quotient rule—two rules covered at an early stage in most basic calculus modules. This approach is adopted by the majority of mainstream mathematics text books and very often forms the basis for mathematics worksheets in Irish higher education. Each worksheet routinely consists of one or two topics and students typically attempt the worksheets immediately after studying the relevant topic(s). By contrast, interleaved practice (also called mixed practice) involves shuffling or interleaving questions on multiple topics so that they appear in a seemingly random order, where consecutive questions relate to different topics. In many Irish higher education exam settings, the topics are interleaved with no hint given to the students as to which method or procedure they must use (CIT, 2018; DCU, 2018; NUIG, 2018). It is therefore prudent for the student to adopt a similar mode of practice to that seen in the exams. One of the principle benefits of interleaved practice is that students must learn to identify the appropriate method for a given problem (Rohrer & Taylor, 2007). Kornell and Bjork (2008) come to the conclusion that the benefits of interleaving are especially useful when it is difficult to discriminate between two similar things. The interleaving effect has been shown to be separate from the spacing effect and also stands without any spacing (Guzman-Munoz, 2017).

There are several possible explanations for the benefits of interleaved practice, the investigation of which is still current in the area of Cognitive and Educational Psychology.
One such explanation is the Discrimination Hypothesis as investigated and described by Wahlheim et al. (2011), which maintains that the main advantage of interleaved practice is that it allows students to compare examples that are similar, but somehow different, by seeing them side-by-side. This aids in the discrimination between categories. If this hypothesis is true, then the best benefit of interleaved practice can be derived when topics are very similar and it is difficult to discriminate between them. This is certainly the case with many areas within mathematics modules in higher education. For example, determining whether to use implicit differentiation or partial differentiation in a first calculus module—two techniques of differentiation with only a subtle difference between the types of expression to indicate which is the appropriate method; or selecting separation of variables or the integrating factor method in a module on methods of solving differential equations—in this case, two integration techniques with only a minor distinction between the formats of the functions dictating which approach should be adopted. Studies carried out by Guzman-Munoz (2017) add further weight to this argument. He also links this hypothesis with working memory capacity (WMC) and found that students with high WMC benefitted slightly more from interleaved practice. He contends that this increased benefit is driven by the fact that in order to compare and contrast questions the information about previous questions must be held in working memory.

The use of interleaved practice in mathematics was examined by Patel, Liu and Koedinger (2016) and also by Mayfield and Chase (2002). The former compared the learning of students who completed blocked practice with those who completed interleaved practice and found evidence that the interleaved practice yielded superior learning outcomes, with mean test results of 79% for students who followed interleaved practice compared to 68% for blocked practice. The authors conclude that in addition to learning how to do the mathematics in
question they also learn how to decide when to carry out different approaches. Mayfield and Chase go further still to conclude that interleaved practice improves students’ ability to problem-solve novel tasks related to the constituent skills practiced. In many higher-level mathematics courses one of the key skills required is how to decide on an appropriate method. Consider again a calculus course that covers both differentiation and integration, and several standard approaches to each. The ability to discern which approach to adopt in a situation is as much a key skill as the procedural mathematics itself, as without this ability the student cannot successfully navigate the problem.

Despite its benefits, interleaved practice is not without its disadvantages. The extra cognitive demand placed on students in selecting the appropriate method for each question results in a slower progression through the questions than in conventional blocked practice. This may have a demotivating effect on students, as it is natural to associate the quality of one’s learning with fluent progress through the questions (Logan, Castel, Haber, & Viehman, 2012). In addition, Dunlosky et al. (2013) review several studies showing that students following mixed practice typically perform significantly worse (i.e. answer fewer questions correctly) in practice sessions such as tutorials, than those undertaking blocked practice. Understandably, students may interpret this poorer initial performance as a measure of their actual learning. However, if one considers these practice sessions in the context of retrieval practice, the findings of Grimaldi and Karpick (2012) that memory is enhanced even when retrieval attempts fail, support the notion that performance during practice is not an indicator of the students’ true learning progress. Instead, it acts as a foundation from which the students’ knowledge grows while the true learning progress is more accurately reflected in the performance during later (delayed) tests. It is therefore important to manage the students’ judgements of their own learning when adopting interleaved practice.
Judgements of Learning and Mathematics Anxiety

As a person learns they develop a feeling for how well they are learning the material and how well they might recall this information in the future. This prediction of future recall is their judgement of learning (Wahlheim, 2011). People often draw the conclusion that they are learning well by a feeling of fluency that they get while working (Benjamin, Bjork, & Schwartz, 1998). Repeating the same or similar tasks gives a learner the feeling of fluency and hence boosts their judgement of learning. However, these feelings can often be misleading and account for the fact that intuitively both students and teachers can make the error of believing that massed practice—and consequently overlearning—enhance learning, whereas it is more accurate to say that they inflate judgements of learning.

Spacing and interleaving both decrease fluency, and as such, each of these techniques can be seen as a desirable difficulty—an approach that creates some new level of difficulty for the learner but is beneficial in the long-term. As a result of this decreased fluency, these strategies can have a negative effect on the students’ judgements of learning. Specifically, they may contribute to students making more mistakes in the learning stage and the longer-term benefits to learning may not be immediately apparent to the learner. This is borne out in much of the literature. Kornell and Bjork (2008) found that massed practice gives learners the impression that their learning is better even when they are presented with test results that show them otherwise. Guzman-Munoz (2017) also found that throughout the experiments students consistently underestimated the beneficial effects of interleaved practice and favoured massed practice. These inaccurate judgements of learning may result in students choosing inappropriate learning strategies and losing some confidence in their abilities.
There may exist some ways in which students’ judgements of learning can become more accurate. Logan et al. (2012) discuss the concept of students’ judgements of learning and how they are adversely affected by the use of spaced practice over massed practice. They investigated a number of possible routes to help students develop more accurate judgements of learning, and in particular to see the benefits of spaced practice. They considered the effects of giving feedback, students gaining experience, and providing brief instruction on the benefits of spaced practice. The biggest impact on judgements of learning was achieved by providing brief instruction on the benefits of spaced practice. When the benefits of spaced practice were pointed out to the students, there was a 10% improvement in the students’ judgements of learning.

Judgements of learning while working through assigned worksheets could impact on students’ emotional states and their feelings about the mathematics module they are studying. Mathematics anxiety was described by Spicer (2004) as “an emotion that blocks a person’s reasoning ability when confronted with a mathematical situation” (p. 1). Being cognisant of the prevalent levels of mathematics anxiety among mature students (Marshall, Staddon, Wilson, & Mann, 2017) and the fact that the level of mathematics anxiety among fifteen year-old students in Ireland is significantly above the OECD average (Perkins & Shiel, 2016), it may be necessary to take some steps to allay any negative effects that the introduction of spaced and interleaved practice may have on the confidence of the student and on their anxiety levels.

A simple strategy to combat mathematics anxiety recommended by Marshall et al. (2017) is to provide notes and resources to students ahead of time on a virtual learning environment. This strategy could be extended to the current context by giving students access to the
worksheets in advance of tutorials. Another recommendation from Marshall et al. (2017) was to give students a mechanism to ask questions either anonymously or out of the spotlight. A digital backchannel (Du, Rosson, & Carroll, 2012) could provide such a means of communication and may be worth considering. Another element shown to combat mathematics anxiety is feedback (Simzar, Martinez, Rutherford, Domina, & Conley, 2015). Ideally, students would get constructive and timely feedback from the lecturer or tutor on their progress with the worksheets. This may help to counteract the effects of the desirable difficulties deliberately introduced into the design of these worksheets.

One further recommendation from Perkins and Shiel (2016) is that students should be provided with activities they can complete successfully. Given the negative impacts that spaced and interleaved practice may have on the judgements of learning of students and their possible impact on the levels of mathematics anxiety of the students, the above recommendation is valuable and could be implemented by including two or three “visibly easy” questions in each worksheet. To describe them as visibly easy is to convey the idea that when a student quickly scans the worksheet, they can see that scattered among the more challenging questions are a few that they immediately know how to do. Such questions would be selected from some of the easiest types of questions on the topic. It is hoped that this may allay some of the negative feelings involved by introducing desirable difficulties.

**Recommendations**

Taking into consideration the benefits of spaced and interleaved practice over massed and blocked practice on learning, as proposed by the literature reviewed here, this section sets out a number of recommendations on how to utilise these strategies effectively, while attempting to offset any negative impacts associated with their introduction.
In terms of the overall structure of a mathematics worksheet we would propose, in line with Rohrer and Taylor (2006), that a worksheet should begin with no more than two or three questions related to the topic(s) most recently covered. The aim here is to allow the student to practise the topic(s) most recently introduced, thereby developing a mastery of new techniques, while at the same time avoiding overlearning. The remainder of the worksheet should consist of a further six to ten questions, where one or two questions from the most recent topic(s) are interleaved with questions selected from previously covered topics. The interleaved aspect of this approach improves learning and promotes the development of the students’ abilities to correctly identify the appropriate method for the relevant problem, as concluded by Rohrer and Taylor (2007) and Kornell and Bjork (2008), while the spacing effect—a strategy which has consistently been found to have a significant positive impact on long-term learning and rated by Dunlosky et al. (2013) as having high utility relative to other learning techniques—is exploited by including questions on previously learned material. Restricting the total number of questions on the worksheet to somewhere between nine and fourteen is intended to ensure that the students are not discouraged by the volume of questions to be attempted.

In order to alleviate the potential increase in mathematics anxiety associated with the introduction of spaced and interleaved practice, we adopt the recommendation by Perkins and Shiel (2016, p.6) of “providing activities that students can complete successfully” and propose that two or three visibly easy questions be scattered throughout the worksheet.
Furthermore, in order to promote the strategies of spaced and interleaved practice, some means of improving students’ judgements of learning should be considered. In light of the findings of Logan et al. (2012), where informing students of the benefits of spaced practice was found to result in the greatest improvement in students’ judgements of learning, we recommend that each worksheet contain some brief information regarding the benefits of both spaced and interleaved practice on learning. A sample text adapted from Logan et al. (2012, p.188) might read as follows: *You may notice that some of the questions on this worksheet relate to work done in previous weeks and that the topics are all mixed together. Research has shown that repeating material you learned in the recent past is good for memory. Mixing questions from related topics also has benefits for your learning and memory. These two things combined can have a powerful effect on your memory of the material covered in these worksheets. However people usually underestimate these benefits. Keep these benefits in mind as you work through the worksheets for this module.*

Where appropriate, other safeguards against mathematics anxiety should be employed e.g. providing questions ahead of time, timely feedback, and a digital backchannel.

**Conclusion**

The aim of this paper was to summarise clearly the benefits of spaced and interleaved practice, to consider any disadvantages that might arise from implementing these practices, and to distil this knowledge into recommendations on how these strategies might be incorporated into the mathematics worksheet in higher education. The difficult role of the educator in the Irish higher education system is to achieve long-term learning in a brief 13-week module while contending with significant constraints on resources and time. Overlearning, massed and blocked practice have each been shown to be ineffective means of achieving long-term retention and yet are still the mainstay of mathematics practice in both
textbooks and instructional materials. Spaced practice is widely accepted to have significant advantages over massed practice in many fields of learning and specifically in mathematics. Interleaving mathematics problems of different types has been found to be particularly effective in increasing long-term retention of material and, crucially, in improving students’ ability to discern which method to apply in different situations. The desirable difficulties introduced by spaced and interleaved practice could have an adverse effect on students’ perceptions of their progress and the authors would strongly recommend that some safeguards, along the lines of those outlined in the previous section, be put in place to limit such negative impacts. In summary, if one was to take into account the recommendations described above when designing a higher education mathematics worksheet, it would arguably look very different to the current standard.

Computer Aided Assessment (CAA) would seem like a comfortable fit with the objectives of this paper and is worth considering as an area of future work. The creation of spaced and interleaved worksheets could be automated, thereby alleviating any extra workload on lecturers. CAA would also facilitate immediate feedback to students and so would hopefully assist in mitigating the negative effects of mathematics anxiety. One possibility would be to use a Virtual Learning Environment/Learning Management System, similar to that used in the Key Skills initiative in the Institute of Technology Tallaght (Marjoram, Moore, O’Sullivan, & Robinson, 2008; Marjoram, Robinson, O’Sullivan, & Carr, 2013).

Given the strong evidence for learning benefits offered by both distributed practice and interleaved practice, the authors hope to assess the effectiveness of the recommendations put forward in this paper on the retention of material for first year engineering students.
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