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# Modeling same-direction two-lane traffic for bridge loading

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## ABSTRACT

Many highway bridges carry traffic in two same-direction lanes, and modeling the traffic loading on such bridges has been the subject of numerous studies. Different assumptions have been used to model multiple-presence loading events, particularly those featuring one truck in each lane. Using a database of weigh-in-motion measurements collected at two European sites for over 1 million trucks, this paper examines the relationships between adjacent vehicles in both lanes in terms of vehicle weights, speeds and inter-vehicle gaps. It is shown that there are various patterns of correlation, some of which are significant for bridge loading. A novel approach to the Monte Carlo simulation of such traffic is presented which is relatively simple to apply. This is a form of smoothed bootstrap in which kernel functions are used to add randomness to measured traffic scenarios. It is shown that it gives a better fit to the measured data than models which assume no correlation. Results are presented from long-run simulations of traffic using the different models and these show that correlation may account for an increase of up to 8% in lifetime maximum loading.

**Keywords:** Bridge; traffic loading; Monte Carlo simulation; bootstrap; kernel density estimators; correlation

## 1. INTRODUCTION

Much work has been done on modeling bridge loading due to two-lane same-direction traffic. In the work by Nowak [1], a number of simplifying assumptions were made – for example that one in 15 heavy trucks has another truck side-by-side, and that for one in 30 of these multiple truck events, the two trucks have perfectly correlated weights. A heavy truck was defined as one with a gross vehicle weight (GVW) in the top 20% of measured truck weights. It was calculated that the maximum load effect in 75 years is

caused by two trucks side-by-side, with each truck having a GVW of 85% of the maximum individual GVW in 75 years. As Kulicki et al. [2] note, the assumptions used were based on limited observations, and the assumptions on weight correlation were entirely based on judgment, as almost no data were available. Moses [3] presents a simple traffic model for estimating multiple presence probabilities as a function of average daily truck traffic (ADTT), and then selects conservative values, some being based on subjective field observations, for calibrating load factors for bridge assessment. Sivakumar et al. [4] refine the definition of side-by-side events to include two trucks with headway separation of  $\pm 18.3$  m (60 ft), and also consider the influence of the bridge length. Sivakumar et al. [5], citing Gindy and Nassif [6], extend this further by classifying multiple-presence events as side-by-side, staggered, following or multiple. They present statistics, derived from weigh-in-motion (WIM) measurements, for the frequency of occurrence of these events for different truck traffic volumes and bridge spans. They describe a method for estimating site-specific bridge loading which uses multiple-presence probabilities calculated either directly from WIM data or estimated from traffic volumes using reference data collected at other sites. It is assumed that the GVW distribution is the same in both lanes, and that there is no correlation between weights in adjacent lanes. Random multiple-presence loading events of each type are generated by selecting any two trucks from the database of WIM measurements and calculating the resulting bridge load effects. In this way, the distribution of load effects from measured traffic is simulated, and lifetime maximum loading can then be estimated by statistical extrapolation.

In the development of the Eurocode for bridge loading [7], characteristic load effects were estimated by extrapolating directly from results for measured traffic, and also by extrapolating from Monte Carlo simulation of traffic, with each lane being simulated independently [8-10].

Croce and Salvatore [11] present a theoretical stochastic model based on a modified equilibrium renewal process of vehicle arrivals on a bridge and note that while existing numerical models are particularly efficient when single-lane traffic flow is considered, they are unsatisfactory for multi-lane traffic, and have often employed drastic

simplifications. In their model, convolution is used to combine load effect distributions for traffic in multiple lanes.

This study is based on WIM data collected at two European sites, in the Netherlands and the Czech Republic. A detailed analysis of the data reveals that for groups of adjacent vehicles in both lanes, there are patterns of correlation and interdependence between vehicle weights, speeds and inter-vehicle gaps. A Monte Carlo simulation model has been developed for evaluating bridge loading due to traffic in two same-direction lanes. This simulation seeks to reproduce the sometimes subtle patterns of correlation that are evident in measured traffic while also adding an element of randomness so as to vary the loading. This study focuses on short to medium span bridges, up to 45 m long, where free-flowing traffic with dynamics is taken to govern [8,12]. The approach described could also be applied to long span bridges if sufficient data on the traffic patterns in congested traffic were available.

## **2. WIM DATA**

The WIM data used as the basis for this study were collected at two sites – at Woerden in the Netherlands, and at Sedlice in the Czech Republic, as detailed in Table 1. The data were filtered to identify unreliable values and photographic evidence from the Netherlands was used to support this data cleaning. Vehicle records were rejected using the following criteria:

- Speed less than 40 km/h or greater than 120 km/h
- GVW less than 3.5 t
- Number of axles recorded as zero or one
- Sum of axle weights different from GVW
- Sum of axle spacings different from wheelbase
- Individual axle weight greater than 40 t
- Individual axle spacing less than 0.4 m
- Individual axle spacing greater than 20 m
- Wheelbase less than 1 m
- Maximum axle load greater than 15 t, and more than 85% of GVW

- Number of axles, axle spacings and axle loads not consistent

Table 1. Summary of WIM data

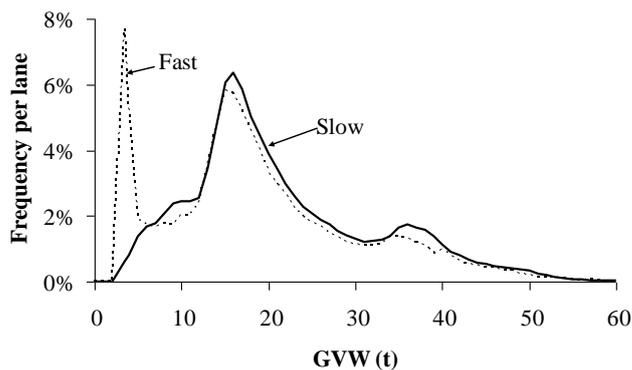
Country	Netherlands (NL)		Czech Republic (CZ)	
Time period	Feb 2005 to June 2005		May 2007 to May 2008	
No. of valid days <sup>a</sup>	77		148	
Time stamp resolution (s)	0.01		0.1	
Vehicles rejected during data cleaning	17 795		6 801	
	<b>Slow lane</b>	<b>Fast lane</b>	<b>Slow lane</b>	<b>Fast lane</b>
Total trucks (after data cleaning)	596 568	49 980	684 345	45 584
ADTT <sup>b</sup>	6 545	557	4 490	261
Maximum GVW (t)	166	75	129	128
No. over 60 t	1 680	36	322	54
No. over 100 t	238	0	10	2

Notes:

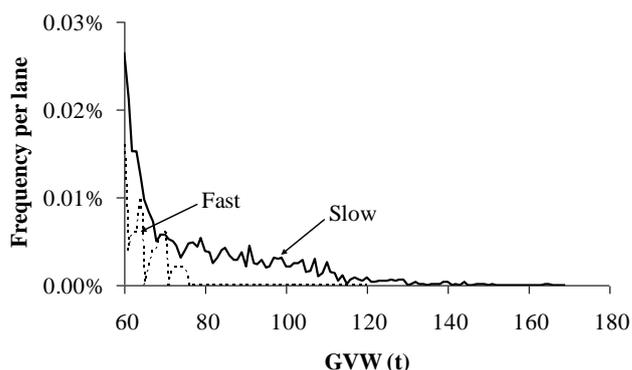
<sup>a</sup> Valid days are weekdays with no interruptions in the measurement record

<sup>b</sup> Average daily truck traffic per lane on valid days

As can be seen from the GVW distributions for each lane in the Netherlands in Fig. 1, there are significant differences between the two lanes, with a much higher proportion of light vehicles in the fast lane (Fig. 1 (a)) and the same is true in the Czech data. In the Netherlands, there is a much higher proportion of extremely heavy vehicles in the slow lane (Fig. 1 (b)) which is important for bridge loading.



(a) Netherlands up to 60 t



(b) Netherlands over 60 t

Fig. 1. GVW distributions

### 3. CORRELATION IN MEASURED DATA

#### 3.1. Vehicle weights

Correlation between weights of successive vehicles can arise from a number of causes. For economic or other reasons, there are times of the day at which heavy vehicles are more likely to travel, and these intra-day patterns cause a low level of correlation within each lane which can be seen by calculating an autocorrelation function. This involves calculating the coefficients of correlation between the weight of each truck (the leading truck) and the truck following it, between the leading truck and the second truck behind it, between the leading truck and the third truck behind it and so on. The results of this are shown in Fig. 2.

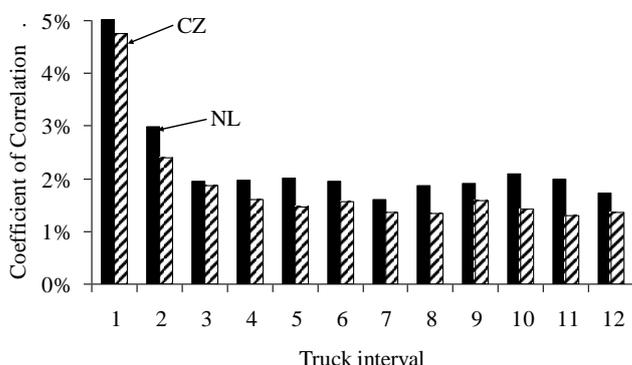


Fig. 2. GVW autocorrelation, slow lane

In the slow lane at both sites, there is an underlying level of correlation of about 2%, but of particular interest is that there is a significantly higher level of around 5% for pairs of trucks (interval of 1). This may be due to driver behaviour whereby similar trucks may tend to form platoons, or because groups of associated vehicles may choose to travel together, and there is much photographic evidence of this at the site in the Netherlands – for example mobile cranes are often accompanied by vehicles carrying ballast. Similar patterns are evident in the fast lane, with an underlying level of correlation of 7.4% and a pairwise correlation of 9.4% in the Netherlands. In the Czech Republic the corresponding figures are lower, 1.4% and 2.2% respectively.

For short to medium span bridges, loading events featuring one truck in each lane (either side-by-side or staggered) are particularly important. To assess if there is any dependence between the weights of these vehicles, each fast-lane truck in the measured data is notionally paired with the nearest truck in the slow lane, and the gap is measured in seconds between the front axles of the two vehicles. The gaps are binned in intervals of 1 s, and average GVWs in both lanes are calculated for each bin. These average GVWs are plotted against the inter-lane gap for the Netherlands in Fig. 3. At both sites, most fast-lane trucks are within 2 seconds of a slow-lane truck – 75% in the Netherlands and 72% in the Czech Republic – and this is evident from the distribution of inter-lane gaps as shown in Fig. 3. There is a significant peak in the average fast lane GVW when the gap is around zero – i.e. when the trucks are very close – and a similar pattern is evident in the Czech Republic. It appears that a heavy truck in the fast lane tends to be associated with a nearby truck in the slow lane, i.e. it is passing another truck. This is just one way of illustrating the patterns of dependence in multi-lane traffic, and the simulation approach presented here successfully reproduces this pattern. The coefficient of correlation can also be calculated for the GVWs of all inter-lane pairs of trucks. In the Netherlands, this has a value of 5.2%, and in the Czech Republic it is 1.5%.

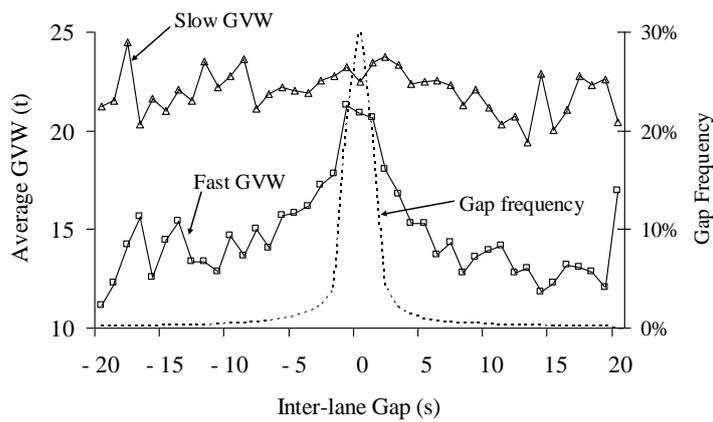


Fig. 3. Inter-lane GVW correlation, and inter-lane gap distribution, the Netherlands  
 The frequency of occurrence of multiple presence bridge loading events can also be calculated from the measured data for bridges of different length. For example, the frequency of occurrence of the two-truck loading event, with one truck in each lane (either side-by-side or staggered), is shown for both sites in Fig. 4. For comparison, values for U.S. traffic are shown for annual daily truck traffic of 2500 to 5000 trucks per day (taken from Sivakumar et al. [5]).

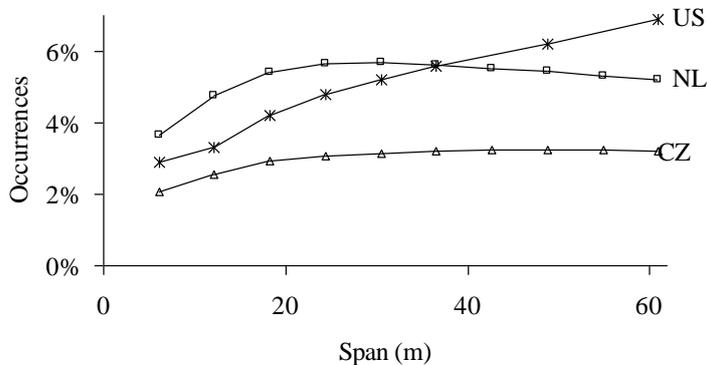


Fig. 4. Multiple presence frequencies – two-truck event

### 3.2. Gaps and speeds

It is well established that the distribution of same-lane gaps between vehicles varies with traffic flow rate [13]; in general gaps are less for higher flows. It is evident from the WIM data used here that there is also some slight dependence between gaps and GVW [20], and that successive gaps are not independent. At both sites, the axle to axle gap observed behind vehicles tends to increase as the GVW increases. This can be attributed partly to driver behaviour, perhaps greater overhang (axle to bumper)

distances, and also to the fact that many trucks in excess of the normal legal weight limit are followed by escort vehicles. The idea that successive gaps are not independent is reasonably intuitive. The platooning effect commonly observed on highways means that smaller gaps tend to occur in groups. Table 2 shows the probabilities of occurrence of gaps less than 2 seconds for three different flow rates in the measured data. For each flow rate, two probabilities are shown – the probability that any gap is less than 2 seconds, and the conditional probability of the gap behind a truck (“Gap<sub>2</sub>”) being less than 2 seconds given that the gap in front of the truck (“Gap<sub>1</sub>”) is also less than 2 seconds. It can be seen that the conditional probability is higher in all cases.

Table 2. Gap probabilities for different flow rates

Flow rate (trucks/hour)	Netherlands			Czech Republic		
	100	200	300	100	200	300
P{Gap ≤ 2 seconds}	5.1%	11.1%	15.2%	5.9%	12.5%	19.6%
P{Gap <sub>2</sub> ≤ 2   Gap <sub>1</sub> ≤ 2}	6.6%	11.6%	17.2%	7.6%	14.7%	22.3%

As might be expected, there is a tendency for heavier vehicles to travel at slightly lower speeds, although most extremely heavy vehicles are travelling at around 80 km/h which would be regarded as a normal highway speed for any truck. Speeds of successive vehicles in the same lane show a relatively high degree of correlation when the inter-vehicle gaps are small, with an average coefficient of correlation for both sites of 53% when the gap is less than 2 seconds. This drops to 15% when the gap is more than 2 seconds.

#### 4. SIMULATION OF TRAFFIC

It is evident from the foregoing that there are discernible patterns in the measured traffic that may be significant for bridge loading. Using measured traffic to calculate a distribution of load effects and then extrapolating from this to lifetime maxima implicitly incorporates the patterns in the traffic, but suffers from high uncertainty due to the extrapolation process. Variation in results from extrapolation of up to 33% have been reported by Gindy and Nassif [14], and up to 20% for the estimation of characteristic load for the Eurocode [9]. The approach used here is to build a Monte

Carlo simulation model that incorporates the patterns and then to run the simulation for a sufficiently long time period to reduce the variance of the estimates from the model. It should be noted that this approach does not eliminate the uncertainty inherent in basing estimates of lifetime maxima on data collected over relatively short periods of time. Seasonal variations that may be present at each site may not have been fully captured, particularly at the Dutch site where the data collection spans only five months. Seasonal variations in flow rates, if present in the data, are modelled by fitting a Weibull distribution to the daily flow rates [20] which is then used in the simulation process to generate variable daily traffic flows.

There are well-established ways of modeling dependence between variables in Monte Carlo simulation. The correlation matrix for a set of variables can be estimated from the measured data, and using the technique described by Iman and Conover [15], random values can be generated for each variable so that both the marginal distribution of each variable and the correlation structure are reproduced in the simulation. A limitation of this is that the correlation matrix is constant, and therefore the correlation between any two variables is assumed to be fixed for all values. This does not model, for example, the observed pattern whereby correlation between weights of successive trucks varies with the weights of both. A more complex correlation structure can be modeled using copula functions [16] and these are widely used in financial markets. In the field of bridge loading, copulas have been used by Sriramula et al. [17] and Srinivas et al. [18] to model dependence between axle weights and spacings on vehicles.

The spatial layout of vehicles on a two-lane bridge can be described by three gap distributions – in-lane gaps for each of the two lanes and inter-lane gaps. The standard approach to simulating random variables is to generate values from the required distributions. In this case, the three gap distributions cannot be simulated independently – for example generating random values from the two in-lane gap distributions will position vehicles in each lane, and this automatically determines the inter-lane gap distribution. For bridge loading, it might be reasonable to assume that the slow-lane and inter-lane gaps are more important than the fast-lane gaps. On this basis the slow-lane and inter-lane gaps can be simulated directly from the distributions, and a good match between observed and simulated gaps can be obtained. However, the simulation of the

fast-lane gaps is completely wrong, with the platooning effect in that lane being lost in the simulation.

In order to build a conventional simulation model for two same-direction lanes, all significant patterns in the measured data must be identified and quantified in some way that can be incorporated into the simulation. It is possible to build a reasonably accurate model in this way, but the process is very site-specific and time-consuming and the model needs to be carefully calibrated. Extending such a model from two to three or more lanes would be very challenging. An alternative multi-dimensional smoothed bootstrap approach is adopted here which avoids many of the difficulties associated with the conventional approach, and in principle can quite easily be extended to more than two lanes.

The principle of bootstrapping is to repeatedly draw random samples from the observed data [19]. In this case, the samples used are “traffic scenarios”, with each scenario consisting of between five and eight slow-lane trucks in succession, with any adjacent fast-lane trucks. In preparation for simulation, the WIM data are analysed and all scenarios are identified. The parameters recorded for each scenario are flow rate, gaps, GVWs and speeds. The flow rate is represented by the number of slow-lane trucks in the current hour, rounded to the nearest 10 trucks/hour. The gaps needed to define the scenario are the gaps within each lane, and one inter-lane gap which positions the first fast-lane truck relative to the leading slow-lane truck in the scenario, as shown in Fig. 5.

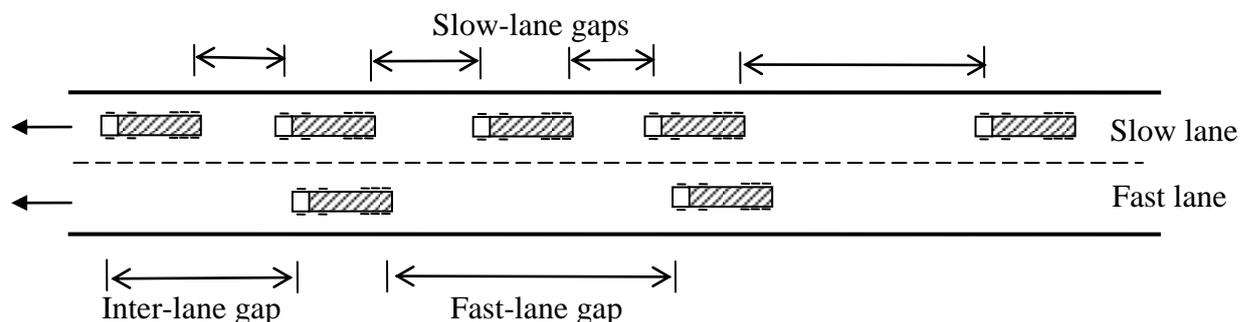


Fig. 5. Traffic scenario

The number of parameters needed to describe a single scenario (i.e. the dimensionality of the problem) varies with the size of the scenario, but in the typical scenario shown in Fig. 5, a total of 21 different parameters are needed – the GVWs and speeds of seven

trucks, six gap values and a flow rate. Correlations between parameters are implicitly included in each scenario.

The aim in setting up the scenarios is to keep them reasonably small so as to maximise the variability in the simulation, but also to have them large enough to capture patterns that may be significant for bridge loading. In order to preserve any significant groups of heavy vehicles in the slow lane, the first and last slow-lane trucks are required to be less than 30 t. Hence, starting from a truck less than 30 t, trucks are included until another less than 30 t is found. The last truck in each scenario becomes the first in the next scenario. In order to provide greater coverage of different scenarios, four scans are made through the WIM data with the minimum scenario size varying from five slow-lane trucks for the first scan up to eight for the last scan, as illustrated in Fig. 6. For example, scenario “S1” starts with the first (28 t) truck in the slow lane, and as the fifth truck (38 t) in this lane weighs more than 30 t, the scenario is extended to include the next truck (12 t). Two fast-lane trucks are also included in S1. As another example, scenario S6 is constructed during the third scan through the data and comprises seven slow-lane and three fast-lane trucks. For this scan, the minimum number of slow-lane trucks per scenario is set to seven, and as the seventh slow-lane truck in S6 has a GVW of 26 t, the scenario ends here and the next scenario (S7) begins.

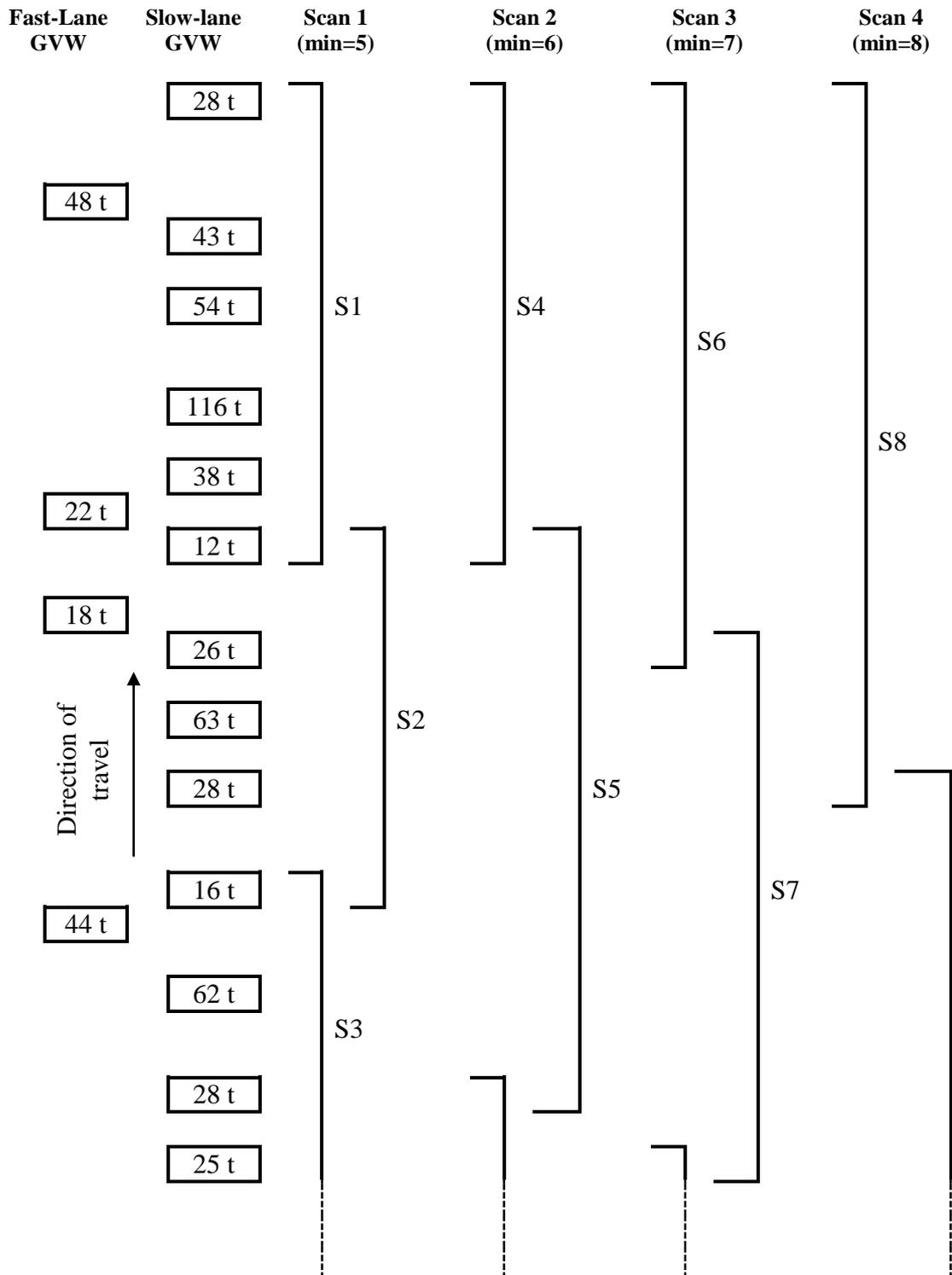


Fig. 6. Identification of different traffic scenarios

In the simulation process, a flow rate is determined for the time of day, based on average measured values for all weekdays. A scenario is selected at random from all

scenarios corresponding to this flow rate. For a given traffic flow rate, each scenario has an equal probability of selection, and this means that the measured relative frequencies of the parameters defining the scenarios are reproduced in the simulation. The number of different scenarios for a given flow rate depends on the quantity of measured WIM data, but at both sites there are in excess of 20 000 scenarios for each of the commonly observed flow rates. The trucks in the selected scenario are added to the stream of traffic, the time is advanced, and another scenario is selected. The scenarios are joined together by overlapping the last truck of the previous scenario with first truck in the new scenario and then discarding the latter. As noted already, the overlapping trucks are all less than 30 t.

This bootstrap process would be expected to produce bridge loading very similar to the measured traffic. The measurements have been collected over a number of months, but in order to estimate lifetime maximum bridge loading, many years of traffic must be simulated. A key part of this process is to extend the simulation to incorporate scenarios that have not been directly observed. Of particular interest is the modeling of vehicles heavier than, and with more axles than, any measured vehicles. Different gap combinations than those observed also need to be allowed to occur. Variations from the observed scenarios are introduced in a number of ways. Each time a scenario is selected in the simulation, the GVWs, gaps and speeds that define it are modified using variable-bandwidth kernel density estimators, as described in the following section. When a GVW has been selected for a particular vehicle, the number of axles is randomly chosen from the measured distribution for that weight. The axle spacings, and distribution of the GVW to individual axles, are also generated randomly from measured distributions for vehicles with different numbers of axles. The approach used for vehicle modeling is described in more detail by Enright and O'Brien [20].

## **5. KERNEL DENSITY ESTIMATORS**

The term “kernel density estimator” describes the use of kernel functions to provide a better estimate of a probability density function from sample data [21]. A simple histogram gives an estimate of the density at discrete points, but is influenced by the choice of the bin size and origin. Replacing each data point by a kernel function and summing these functions gives a better estimate. Different kernel functions can be used

– they are typically symmetric unimodal functions such as the Normal density function. In Monte Carlo simulation, for each random variable, some estimate of its probability density is required. This estimate can be a parametric fit to the data or some non-parametric density. One non-parametric method is to use interpolation on the empirical cumulative distribution, but using a kernel density estimate gives a better coverage of the design space which is important for generating traffic loading scenarios that will be critical for bridges. As Hormann and Leydold [22] point out, the “smoothed bootstrap” method – re-sampling the observed data and adding some noise – is the same as generating random variates from the kernel density estimate, but without needing to compute the estimated density. In this study, the smoothed bootstrap is applied to three variables – GVW, gaps and speeds. Each value  $x_i$  taken from the observed traffic scenarios is modified by adding some noise:

$$X_i = x_i + K[h(x_i)] \quad (1)$$

where  $K$  is a kernel function, centered at zero with a variable bandwidth  $h$  which depends on the value of  $x_i$ .

As Scott [21] suggests, the choice of which kernel function to use is much less important than the choice of bandwidth. A triangle kernel is used here for gaps because its boundedness is useful at very small gaps, and a Normal kernel is used for GVW. Equivalent Normal and triangle kernel functions are shown in Fig. 7. The bandwidth of the triangle kernel in this example is 1.0, and the bandwidth (standard deviation) for the equivalent Normal kernel is 0.411 [21].

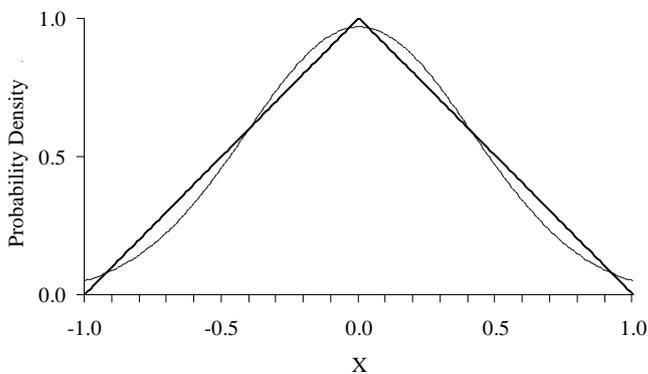


Fig. 7. Normal and triangle kernel functions

There does not appear to be a suitable general theoretical method for choosing the optimal bandwidth. For a Normal kernel applied to a sample of size  $n$  drawn from a Normal population, Scott [21] shows that the mean square error of the density estimate is minimized by using a bandwidth of :

$$h = 1.06 \sigma n^{-0.2} \quad (2)$$

This is of limited use here as the variables (GVWs, gaps and speeds) are not Normally distributed, but it provides an initial estimate of the bandwidth. Scott also discusses adaptive smoothing where the bandwidth of the kernel function is varied and cites the approach developed by Abramson [23]:

$$h_i = \frac{h}{\sqrt{f(x_i)}} \quad (3)$$

where  $f(x_i)$  is the density function.

This approach, adopted for this study, gives relatively small bandwidth at values that occur frequently, and higher bandwidth in the tails where data are sparse and more smoothing is needed. Scott [21] argues that any choice of  $h$  within 15-20% of the optimum will often suffice for estimating densities and suggests starting with an oversmoothed value and reducing the bandwidth until “very local noise near the peaks” is evident. This is the approach that has been adopted here – various bandwidths were tested in simulation and the resulting simulated distribution of each variable was compared with the observed distribution. It is generally quite clear when oversmoothing happens. The physical traffic model also plays a part in selecting a suitable bandwidth structure. It is important not to oversmooth gaps below 2 seconds which are particularly important for bridge loading. Oversmoothing same-lane gaps above 2 seconds has a noticeable adverse effect on inter-lane gaps. The bandwidth used increases linearly up to 2 seconds and is constant above that, as can be seen in the formulae in Table 3 and in Fig. 8(b). A boundary kernel is used for same-lane gaps that are very close to the assumed minimum gap of 0.2 s – when the observed gap is less than  $0.2 + h$  the triangle kernel is shifted so that it is centered at  $0.2 + h$ . The modeling of the upper tail of the GVW distribution is critically important, and O’Brien et al. [24] describe a method

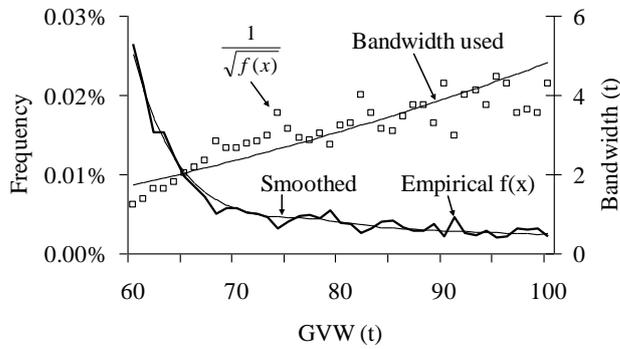
which involves fitting the tail of a Normal distribution to the upper tail of the measured GVW distribution above a selected threshold value. This allows for interpolation between relatively sparse data values and for extrapolation to higher GVW values that are likely to be encountered during the lifetime of a bridge. Using a Normal kernel with a suitable variable bandwidth achieves a GVW distribution in the simulation which matches the tail fitted to the measured data, but a bias is found in the yearly maximum load effects because the traffic scenarios which feature the very heaviest vehicles tend to be over-represented. To overcome this bias, when a GVW above the threshold value (100 t in the Netherlands, 62 t in the Czech Republic) is selected as part of a traffic scenario, it is replaced by a random value generated from the fitted Normal tail. The chosen bandwidth formulations for the different parameters are summarised in Table 3. Fig. 8 illustrates typical bandwidth structures used for GVWs and gaps. The empirical frequencies  $f(x)$  are also shown, together with the distribution which results from the smoothing, and re-scaled values of  $1/\sqrt{f(x)}$ .

Table 3. Kernel bandwidths

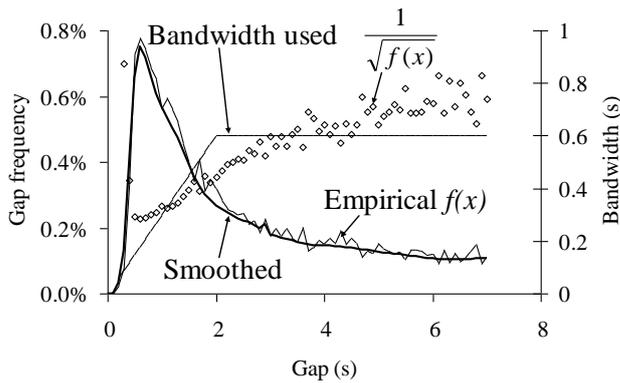
<b>Variable</b> ( $x$ )	<b>Kernel</b>	<b>Bandwidth</b>
Slow-lane GVW (t) <sup>a</sup>	Normal <sup>b</sup>	$0.08 \frac{x^2}{Max(x)}$
Fast-lane GVW (t)	Normal	$0.065 \frac{x^2}{Max(x)}$
Slow-lane gap (s)	Triangle	$\min(0.2x, 0.4)$
Fast-lane gap (s)	Triangle	$\min(0.3x, 0.6)$
Inter-lane gap (s)	Triangle	$\min(0.08 x , 0.16)$
Slow-lane speed (km/h)	Triangle	0.6
Fast-lane speed (km/h)	Triangle	1.0

Notes: <sup>a</sup> For GVWs,  $Max(x)$  is the site-specific maximum observed GVW per lane

<sup>b</sup> The kernel bandwidth is used up to a site-specific threshold GVW value; above this the tail of a Normal distribution is used.



(a) Slow lane GVW – the Netherlands



(b) Fast lane gaps – the Netherlands

Fig. 8. Bandwidth structure

An example of oversmoothing the speed distribution is shown in Fig. 9. The speed distribution is not Normal, but an application of equation (2) suggests a theoretical bandwidth in the region between 0.7 and 0.9 s, depending on how much of the tails are included.

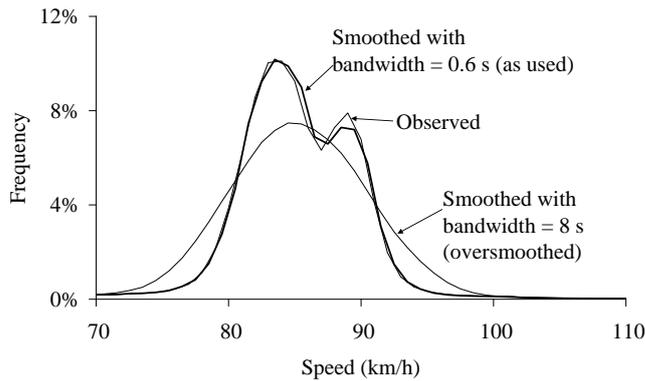


Fig. 9. Speed oversmoothing (slow lane, the Netherlands)

## 6. VALIDATION

In order to assess the simulation models, comparison is made between bridge loading by measured traffic and by simulated traffic on bridges of different lengths – 15, 25, 35 and 45 m. For the measured traffic, bridge load effects are calculated by moving the measured stream of traffic over each bridge. For convenience, these are referred to in the following as “measured” load effects. Daily maximum values are calculated for three load effects – mid-span bending moment on a simply supported bridge (LE1), support shear at the entrance to a simply supported bridge (LE2), and for bridges which are 35 m or longer, hogging moment over the central support of a two-span continuous bridge (LE3).

As well as calculating the overall daily maxima, different loading event types are analysed. It is evident that the two most important loading events in the lifetime maximum loading for the spans considered are the one-truck event (“1+0”) and the two-truck event with one truck in each lane (“1+1”). As the span increases, four other event types are included in the comparison of the different simulation methods – the 1+2, 2+1, 2+0 and 2+2 events, where “ $i+j$ ” indicates  $i$  and  $j$  truck(s) in the slow and fast lanes respectively. These are less onerous for the spans considered at the two sites, but could become significant at longer spans or at other sites with different traffic characteristics. The 1+2 and 2+1 events are considered for spans of 25 m and longer, the 2+0 event for the 35 and 45 m spans, and the 2+2 event for the 45 m span. A summary of loading event types considered is given in Table 4.

Table 4. Bridge loading event types

<b>Event</b>	<b>Description</b>
1+0	One truck in the slow lane
1+1	One truck on the bridge in each lane, side by side or staggered
1+2	One truck in the slow lane with two trucks in the fast lane
2+0	Two trucks in the slow lane (“following”)
2+1	Two trucks in the slow lane with one in the fast lane
2+2	Two trucks in each lane

To assess the effects of correlation, an uncorrelated simulation model was also developed in which GVWs, slow-lane gaps, and speeds are drawn independently for each truck from the observed distribution in the appropriate lane. Gap distributions are measured at 25 different flow rates, and the distribution appropriate to the flow (time of day) is used. For a site-specific percentage of slow trucks, a fast-lane truck is generated and positioned relative to the slow-lane truck by drawing a value from the inter-lane gap distribution. As noted earlier, this does not model the fast-lane gaps well

For comparison purposes, the two simulation models – smoothed bootstrap and uncorrelated – were run for 2000 days, and the simulated and measured results plotted on Gumbel paper. This is a re-scaled cumulative distribution function on which the Gumbel extreme value distribution appears as a straight line [25]. An example is shown in Fig. 10 for 1+1 events on a 35 m bridge in the Netherlands, and this illustrates that the smoothed bootstrap gives a significantly better fit to the measured data.

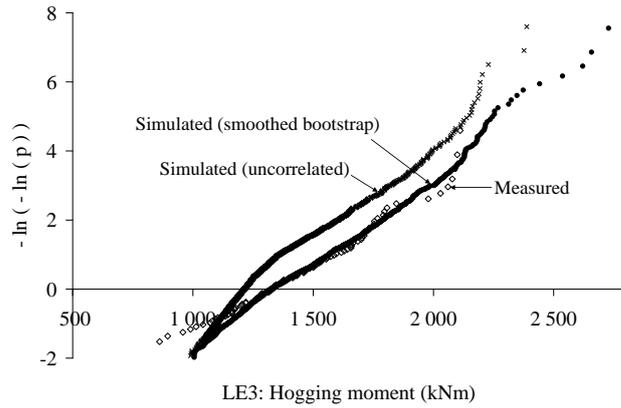


Fig. 10. Simulated and measured daily maximum load effects

A least squares measure is used to quantify the goodness of fit of the two simulation models to the measured load effects. As suggested in Castillo [26], the deviation is measured in terms of load effect value – i.e. in the x direction rather than the more usual y direction. In order to compare all results, a normalised least squares score is computed as:

$$L = \frac{\sqrt{\frac{\sum_{i=1}^{N_{obs}} (obs_i - sim_i)^2}{N_{obs}}}}{obs_{max}} \quad (4)$$

where:

$N_{obs}$  is the number of days in the observed data

$obs_i$  and  $sim_i$  are corresponding values of observed and simulated daily maximum load effect values

$obs_{max}$  is the largest observed daily maximum load effect value over all days

As 2000 days are simulated, there are many more simulated daily maxima than observed. For each observed value, the corresponding simulated point is selected as the value with the closest empirical probability.

The average score is computed for each simulation method for each of the six most important event types at each site. The average is computed over three load effects on the selected spans at both sites. The uncorrelated simulation model is compared with the smoothed bootstrap by calculating the ratios of the average scores, and the results are shown in Table 5. A score greater than 1 indicates that the smoothed bootstrap gives a better fit, in general, to the measured data. As an illustration of these scores, in Fig. 10 the score for the uncorrelated curve relative to the smoothed bootstrap curve is 3.23. Due to the random nature of both measured and simulated loading, scores close to 1 can be interpreted as indicating that both methods match the measured results equally well and, as might be expected, this is the case for the one-truck 1+0 event. Significant differences become apparent in the critically important 1+1 event, and in loading events featuring three or more trucks.

Table 5. Ratios of average scores for goodness of fit.

	<b>Event Type</b>					
	<b>1+0</b>	<b>1+1</b>	<b>2+0</b>	<b>2+1</b>	<b>1+2</b>	<b>2+2</b>
Netherlands	1.08	2.05	1.02	1.37	4.97	1.33
Czech Republic	1.05	1.28	1.04	2.19	2.92	1.11

## 7. RESULTS

To see what effect the different modeling assumptions have on the characteristic maximum loading, both methods were used to simulate 2500 years of traffic. In the Eurocode for bridge loading [7], the value with a 5% probability of exceedance in 50 years is specified for design which is the same as the value with a return period of approximately 1000 years. The focus in the AASHTO design code is on the mean 75-year maximum [27], and the effects of the different models on this are also calculated. Simulation of 2500 years of traffic are used to greatly reduce the variance of the estimates calculated from the model for lifetime maximum loading [20]. These estimates are based on current traffic volumes – no growth in traffic volumes is assumed over the design lifetime of 50 or 75 years.

Lateral distribution is accounted for by applying different lane factors to truck weights in the fast lane. These factors are based on finite element analyses carried out by the authors [20] for concrete bridges. In these analyses, for spans up to 20 m, solid slab decks were assumed; girder and slab construction was assumed for longer spans, with girders at 1 m centers for spans up to 35 m and larger girders at 2 m centers for spans over 35 m. In reality, the contribution of each truck to the maximum stress that occurs in a loading event will depend on the relative weights and positions of the trucks, as well as on the type of bridge. For the purposes of this analysis, two extremes are analysed – high and low lateral distribution. For bending moments on bridges with high lateral distribution, the lane factor used for the fast lane is 1.0 (i.e. no reduction), and 0.45 for low distribution. Maximum shear at the supports occurs when trucks are close to the support, and there is less opportunity for lateral distribution. In this case, a factor of 0.45 represents high distribution, and 0.05 is low. The lane factors and load effects used for the analysis are summarised in Table 6.

Table 6. Load effects and lane factors

Load Effect	Lane Factors (fast lane)	
	Low	High
LE1 Mid-span bending moment, simply supported	0.45	1.0
LE2 Support shear, simply supported	0.05	0.45
LE3 Central support hogging moment, 2-span continuous	0.45	1.0

Sample results are plotted in Fig. 11 which shows simulated annual maxima on a 45 m bridge in the Netherlands with high lateral distribution. Four event types are shown – one truck in the slow lane (1+0), one truck in each lane (1+1), two trucks in the slow lane (2+0), and one truck in the slow lane with two trucks in the fast lane (1+2). For the 1+0 event, both models give the same results, but for events involving two or more trucks there are significant differences between the two simulation models, with the smoothed bootstrap method giving more conservative results than the uncorrelated

model. The curves are reasonably parallel for the 1+1 and 2+0 events, but in the case of the 1+2 event, the curves converge as the return period increases. It can be seen that in this example, the 1+1 event governs at the 1000-year return level.

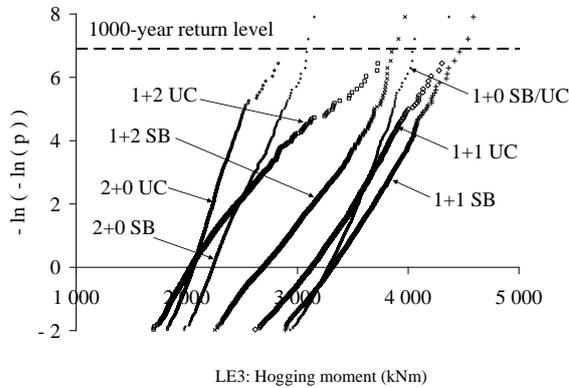


Fig. 11. Annual maxima - smoothed bootstrap (SB) and uncorrelated model (UC)

The increases in characteristic maximum load effects due to correlation in models are summarised in Table 7 for the four bridge lengths and three load effects considered at each site, with all significant differences underlined, using 99% confidence intervals to test for significance. These confidence intervals are estimated for each value using a parametric bootstrap and in general differences between -3% and +3% for the 1000-year values in Table 7 are not significant, although in some cases the confidence interval is slightly larger than this. For the 75-year values, differences between -2% and 2% are generally not significant.. It can be seen that correlation effects can account for an increase in loading of up to nearly 8%, with typical values of around 5%, particularly when lateral distribution is high. The types of loading event that govern the characteristic maximum at the 1000-year return level are also shown in Table 7. In some cases, just one event type is clearly dominant (i.e. either the 1+0 or the 1+1 event), but in other cases there is a mixture of both event types, and for the longer spans (35 and 45 m) in the Czech Republic, some simulated 1+2 events produce bending moments close to the characteristic values.

Table 7. Increase in characteristic maximum load effects due to correlation in models

1000-year return period	75-year mean
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Lane Factors <sup>a</sup>	Site <sup>b</sup>	LE <sup>a</sup>	Bridge length				Governing event type <sup>c</sup>	Bridge length			
			15 m	25 m	35 m	45 m		15 m	25 m	35 m	45 m
High		1	<u>5.9%</u>	<u>5.2%</u>	<u>3.8%</u>	<u>4.5%</u>	1+1	<u>6.7%</u>	<u>6.6%</u>	<u>5.7%</u>	<u>5.0%</u>
	NL	2	0.9%	0.6%	0.7%	0.5%	1+0 / 1+1	0.9%	0.8%	0.9%	0.7%
		3			<u>6.9%</u>	<u>5.4%</u>	1+1			<u>5.0%</u>	<u>4.4%</u>
		1	<u>5.5%</u>	<u>7.8%</u>	<u>6.6%</u>	<u>4.9%</u>	1+1 / 1+2	<u>4.6%</u>	<u>6.4%</u>	<u>6.8%</u>	<u>5.9%</u>
	CZ	2	<u>5.6%</u>	<u>3.9%</u>	2.5%	2.4%	1+0 / 1+1	<u>3.2%</u>	<u>2.4%</u>	2.1%	2.1%
		3			2.1%	4.9%	1+1 / 1+2			2.4%	<u>3.6%</u>
Low		1	1.3%	-1.2%	-1.1%	-1.3%	1+0 / 1+1	0.8%	-0.2%	-0.3%	-0.3%
	NL	2	0.5%	0.0%	0.2%	-0.5%	1+0	0.6%	0.2%	0.4%	0.1%
		3			0.9%	2.7%	1+0 / 1+1			0.6%	1.5%
		1	<u>6.5%</u>	<u>6.1%</u>	<u>4.9%</u>	2.6%	1+0 / 1+1	<u>3.8%</u>	<u>3.6%</u>	<u>3.3%</u>	<u>2.2%</u>
	CZ	2	<u>5.3%</u>	<u>3.5%</u>	1.0%	2.2%	1+0	<u>2.8%</u>	<u>2.0%</u>	1.0%	1.7%
		3			1.1%	0.7%	1+0 / 1+1			0.9%	0.9%

Notes:

<sup>a</sup> Refer to Table 6 for a summary of lane factors and load effects

<sup>b</sup> NL=Netherlands, CZ=Czech Republic

<sup>c</sup> Refer to Table 4 for a summary of loading event types

A closer examination of the events in the simulations that produce the characteristic 1000-year loads shows that for bridges with low lateral transfer, the critical loading event for bending moment is typically an extremely heavy vehicle in the slow lane (80% to 90% of the 1000-year GVW), sometimes with a standard vehicle (in the range 30 to 50 t) in the fast lane – similar to Turkstra’s rule [28]. For bending moment in bridges with high lateral distribution, it is a very heavy vehicle (60% to 80% of 1000-year GVW) in the slow lane with a moderately heavy vehicle (50 to 60 t) in the fast lane – a variation on Turkstra’s rule. For shear at the supports, lateral distribution tends to be

low, and the dominant event type is usually a single extremely heavy truck in the slow lane (75% to 95% of the 1000-year GVW).

## **8. CONCLUSIONS**

There are subtle patterns of correlation evident in measured traffic data. This inter-dependence between weights, speeds and inter-vehicle gaps for adjacent trucks affects the estimation of lifetime maximum bridge loading. While it may be possible to model this dependence reasonably well using conventional Monte Carlo simulation techniques, an alternative multi-dimensional smoothed bootstrap approach is presented here which re-samples observed traffic scenarios and uses kernel functions to introduce additional variation. The traffic scenarios are defined so as to capture patterns that may be significant for bridge loading, and to maximise variability in the simulation. The method is relatively simple to implement for any new site, and could be extended to three or more lanes. It is effectively the same as sampling from empirical distributions (for GVW, gaps and speed), but with correlation and some additional smoothing and randomness. It potentially could be used to model congested or partly congested traffic, if sufficient data were available. The choice of bandwidth for the kernel smoothing functions is somewhat arbitrary, although results for characteristic bridge loading are, within reason, not too sensitive to this choice.

The model presented provides a better fit to measured data across the range of key loading event types than is obtained with a model which does not include any correlation effects. The effects of correlation on lifetime loading may be as high as 8% for the range of bridge spans considered.

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