A non-linear PID controller for CSTR using local model networks

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A Non-linear PID Controller for CSTR
Using Local Model Networks

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Abstract - The basic PID controllers have difficulty in dealing with problems that appear in complex non-linear processes. This paper presents a practical non-linear PID controller that deals with these non-linear difficulties. It utilises a local model (LM) network, which combines a set of local models within an artificial neural network (ANN) structure, to adaptively characterise the process non-linearity. Then a local controller network is formulated through a gating system deduced from the LMN to handle the non-linearity. A continuous stirred tank reaction (CSTR) case study illustrates the practicality of this method in the modelling and control of non-linear processes. PID controllers are still alive and appropriate for the control of non-linear processes.

I. INTRODUCTION

Proportional, integral and derivative (PID) controllers have existed for more than 50 years. The reason why PID controllers have gained such popularity is that the controller can be tuned by means of simple rules of thumb, and detailed knowledge about the system is not necessary.

The basic PID controllers have difficulty in controlling processes with complex nonlinearity. To date, many sophisticated algorithms have been used to help the PID controller work under such difficulties ([1], [2]). Talking about neural networks, generally, there are two different ways of applying a neural network to solve control-engineering problems. One is to use Artificial Neural Networks (ANN) to adjust the parameters of a conventional controller. The other method is the use of the ANN as a direct controller. How are the PID parameter values connected with neural network controllers? Like the two ways to build ANN controllers, there are two ways to make the connection. One method is to adjust the PID parameters by ANN ([3], [4]); the other method is to create the ANN based on the systems output error signal ([5], [6], [7]). The first method involves emulating the thoughts of an expert control engineer by tweaking the tuning parameters according to the empirical rules. Its application is limited and the computation load is normally extensive. The second method is an application of adaptive control, which has been used in industrial application and brings some promising results. However, the selection of the network training data sets is not a trivial problem, and the computational load is highly intensive. So far, most of the approaches lose the simplicity of implementation, which is the most attractive feature of the original PID control approach.

Recent advances in LM networks give a neat extension of the basic PID structure to handle non-linear systems using neural networks. This approach decomposes the system into a number of smoothly overlapping local operating regimes in which a local model describes the local regional properties. The global non-linear model is created by combining the local models through an ANN structure.

In this paper, we have discussed the issue of PID controller application using neural networks. In sections 2 and 3, the construction of local model (LM) networks and local controller (LC) networks are discussed. Then the CSTR case study is presented and performance results are given in section 4. The paper ends with some conclusions and suggestions for future work in section 5.

II. LOCAL MODEL NETWORKS

Local model (LM) networks were first introduced by Johansen and Foss in ([8], [9]) as a means of decomposing non-linear auto-regressive moving average with exogenous inputs (NARMAX) models into an insightful structure for system identification and control. Murray-Smith ([10], [11]) presented further reports on LM network, which presented this approach as one of the standard techniques to combine linear models and ANN to characterise the non-linearity. Figure 1 shows the general structure of this scheme.

Fig.1.  Local Model Networks
We assume that at each time instant, the process behaves in some uniquely characterisable way with each local operating regime \( \Phi_i \), of which we use a function \( f_i \) to describe the property. Then we associate a validity function \( \rho_i \) to determine the validity of the operating regimes given the current operating point \( \tilde{\phi} \). The modelling problem is to robustly estimate the function \( f_i \) from observation data and existing apriori information so as to pre-structure and parameterise the model structure \( \hat{f}_i \). Please refer to ([8] & [9]) for detailed information.

One straightforward and simple approach to the modelling problem is to use a set of linear local models, which is appealing for modelling complex non-linear systems due to its intrinsic simplicity and the weak assumptions required. The linear models can be obtained in several different ways: fitting the parameters of a specified model structure to input/output data obtained from the physical process, fitting the parameters to the simulated response from a detailed fundamental model, or calculating these parameters using differential linearization.

We shall consider the general non-linear state space system, with state vector \( x \) and input \( u \):

\[
\begin{align*}
\dot{x} &= f(x,u) \\
y &= g(x,u)
\end{align*}
\]

Linearization of non-linear dynamic systems of the form of (1) is a standard procedure ([12]). Consider the linearization of \( f(x,u) \) with respect to \( N \) designed operating regimes, these linearized models are created and indexed by \( i \) together with an operating point vector \( \tilde{\phi} \) and \( N \) validity functions \( \rho_i \) as follows ([9]).

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{N} \left[ A_i (x - x^e_i) + B_i u^d_i \rho_i(\tilde{\phi}) \right] \\
y(t) &= \sum_{i=1}^{N} \left[ y^e_i + C_i (x - x^e_i) + D_i u^d_i \rho_i(\tilde{\phi}) \right]
\end{align*}
\]

in which, \( u^d_i = u - u^e_i \), the superscript \( e \) denotes the equilibrium related variable. The state and output of the nonlinear system (1) can be approximately recreated from the \( N \) linear systems of (2).

III. LOCAL CONTROLLER NETWORK

Local controller (LC) networks, the control version of LMN was introduced in [13] and further extended in ([14],[15]). In general, the global control signal is defined by

\[
u(t) = \sum_{i=1}^{nq} C_i(\psi^c(t)\rho_i(\tilde{\phi}(t)) \]

\( C_i \) denotes the local controller for each local model \( f_i \). The \( nq \) local controllers thus obtained are blended using the same validity function \( \rho_i \) which are used in the LMN. The controller information vector \( \psi^c \) consists of past control inputs, current and past plant outputs, and the current and past values of the reference signal \( y_{ref} \). Figure 2 shows a LC network with a gating system. Its basic idea is to adaptively blend various controllers at different operating regions of the process in a proper way through a gating system. The gating system \( \rho_i \) results from the approach formulating the LM network.

IV. CASE STUDY

A. CSTR PROCESS

Continuous stirred tank reactor (CSTR) is a highly non-linear process. A schematic of the CSTR system is shown in Fig.3. A single irreversible, exothermic reaction is assumed to occur in the reactor.
\[ T(t) = \frac{q_i}{V} (T_f - T(t)) + K_1 C(t) \exp \left( -\frac{E}{RT(t)} \right) \]
\[ + K_2 q_c(t) \left[ 1 - \exp \left( -\frac{K_4}{q_c(t)} \right) \right] T(t) - T(t) \]
\[ \dot{C}(t) = \frac{q_i}{V} \left( C_f - C(t) \right) - K_0 C(t) \exp \left( -\frac{E}{RT(t)} \right) \]

where \( q_c(t) \) is the coolant flow rate, \( T(t) \) is the temperature of the solutions and \( C(t) \) is the effluent concentration. The model parameters defined and nominal operating conditions are shown in Table 1. The objective is to control \( C(t) \) by manipulating \( q_c(t) \).

### Table 1. Nominal CSTR Operating Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_i )</td>
<td>100 l/min</td>
</tr>
<tr>
<td>( T_f )</td>
<td>350 K</td>
</tr>
<tr>
<td>( T_c )</td>
<td>350 K</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>1.44*10^13 Kl/min/mol</td>
</tr>
<tr>
<td>( V )</td>
<td>100 l</td>
</tr>
<tr>
<td>( E/R )</td>
<td>104 K</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>0.01 l/mol</td>
</tr>
<tr>
<td>( K_3 )</td>
<td>700 l/min</td>
</tr>
<tr>
<td>( K_0 )</td>
<td>7.2*10^0 min^-1</td>
</tr>
</tbody>
</table>

The CSTR process is with exponential terms and product terms. Its open-loop step tests show that the output concentration responses vary from over-damped to under-damped, indicating the variable dynamics in the CSTR process. Fig. 4 is the step response of concentration output \( C(t) \) when the coolant flow rate \( q_c(t) \) varies from 85 l/min to 110 l/min. The CSTR exhibits highly non-linear dynamical behaviour.

![Dynamic response of the CSTR plant](image)

The difficulty of modelling using LM networks lies in that it requires careful consideration of the following options: the number of the regimes, the variables to be used to define the regimes, and the size and shape of the regimes. We manually decomposed the work regime of the CSTR into N small regimes based on a-priori information, each of which linearly approximates the local property of the assigned regime. Normalised Gaussian function is used as the weighting function in the ANN structure to blend the local models. Simulations were carried out to model the system with from 3 up to 10 models. The global model with 5 local models meets the best tradeoff between the number of the local models and the quality of the performance. The selected 5 local operating regimes are as follows:

\[
C^1 = 1.2980 - 1mol/l, T^1 = 43295K, q^1 = 1.00 e2/min \\
C^2 = 8.5069 - 2mol/l, T^2 = 442K, q^2 = 9.8899e1/l/min \\
C^3 = 5.8541 - 2mol/l, T^3 = 450K, q^3 = 8.8299e1l/min \\
C^4 = 2.9468 - 2mol/l, T^4 = 465K, q^4 = 6.8789e1l/min \\
C^5 = 1.4630 - 2mol/l, T^5 = 481K, q^5 = 0.0438e1l/min
\]

in which \((C_i^1, T_i^1, q_i^1)\) denotes the linearization point of the \(i\)th local model.

We choose a set of step input signal \( q_c(t) \), which varies from 50 l/min to 110 l/min, as shown in Fig.6. It covers the highly dynamic operating area of CSTR process. The LM network outputs are given in Fig.7, which compares the effluent concentration outputs \( C(t) \) and the temperature outputs \( T(t) \) from the CSTR process with the corresponding outputs from the LM network, when the input signal (coolant flow rate) \( q_c(t) \) varies. We can see the goodness of the matching between the LM network model outputs and the process CSTR outputs. It is worthwhile to note that, when the coolant flow rate \( q_c(t) \) = 110 l/min, the CSTR output is highly under-damped, however, the LM network output still
matches the CSTR output with high accuracy, which proves the effectiveness of the proposed LC network.

C. PI/PID CONTROLLER FOR CSTR

The local PI/PID controller parameters were designed firstly using the ‘Zeigler-Nichols ultimate cycle tuning rule’ based on the models developed above, and then empirically adjusting the PID parameters to get optimal performance. The control signal is deduced from

$$u(t) = K_p \left( e(t) + \frac{1}{T_i} \int_{t_0}^{t} e(t) dt + T_d \frac{de(t)}{dt} \right)$$

The PID parameters we use in this paper are as follows:

- $K_{p1} = 2.55 \times 10^3$, $T_{i1} = 9.59 \times 10^{-1}$, $T_{d1} = 0.22$
- $K_{p2} = 3.31 \times 10^2$, $T_{i2} = 5.12 \times 10^{-1}$, $T_{d2} = 0$
- $K_{p3} = 4.042 \times 10^2$, $T_{i3} = 2.67 \times 10^{-1}$, $T_{d3} = 0$
- $K_{p4} = 2.08 \times 10^2$, $T_{i4} = 4.34 \times 10^{-1}$, $T_{d4} = 0$
- $K_{p5} = 2.01 \times 10^4$, $T_{i5} = 5.35 \times 10^{-1}$, $T_{d5} = 0$

The global non-linear PID controller is formulated by blending the local controllers through the gating system resulted from the LMN structure. The behaviour of the non-linear PID controller is shown in Fig.8, from which we can see the smooth transient response when the set-point $C(t)$ changes from 0.02 mol/l up to 0.13 mol/l. It should be noted that the set-point $C(t)=0.13$ is very close to the unstable region of CSTR process, in which there is only small overshoot. It validates the LC network developed based on the LM network.

Moreover, the global performance of the LC network highly depends on the performance of the local controllers. The gating system, as a weighting function, smooths the transient response when the set-point changes.

Fig. 6: Comparison of the CSTR process output and the LM network output
Solid line is from the plant, dash-dotted line is from the LM networks.
Fig. 7. CSTR step responses of the temperature $T(t)$ and of the effluent concentration $C(t)$ in closed loop. Solid line is from the LN controller; dashed line is the input.

V. CONCLUSION

We present a non-linear PID controller using LMN and LCN in this paper. This paper illustrates the simplicity and practicality of the method in the identification and control of non-linear system CSTR (Continuous Stirred Tank Reactor). It proves that PID controllers are still alive and appropriate under non-linear difficulties. Concerning the difficulty of manually tuning PID parameters, further work will focus on auto-tuning PID controller strategies as well as stability issues for non-linear process control based on the LM Network.

VI. REFERENCES


