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Implications of the Hall Effect for Turbulent Molecular Clouds

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Implications of the Hall effect for turbulent molecular clouds

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Abstract. We present the first study of turbulent decay in systems in which the Hall effect is important. We use a new, stable, explicit algorithm to integrate the multifluid MHD equations in the presence of the Hall effect.

We find that in Hall-dominated systems effect introduces more structure in density on all length scales, and marginally reduces the turbulent energy decay rate, than in equivalent systems which are ambipolar dominated.

1. Introduction

Turbulence is believed to be an important factor in determining the evolution of molecular clouds, both in terms of their global behaviour (such as collapse) and also in terms of the initial mass function of the stars formed in such clouds. It is difficult to make progress analytically with studies of turbulence in a medium such as a molecular cloud. Many authors (e.g. Mac Low et al. 1998; Mac Low 1999; Ostriker et al. 2001; Vestuto et al. 2003; Gustaffson et al 2006; Glover & Mac Low 2007) have investigated the evolution of magnetohydrodynamic (MHD) turbulence, usually in the case of ideal MHD but sometimes also incorporating ambipolar diffusion, sometimes called "ion-neutral friction".

It is important to understand how turbulence in molecular clouds decays as this provides a constraint on the injection of energy into the cloud, for example from star formation, to maintain the turbulence at observed levels. For hydrodynamics it is well known that the kinetic energy in the fluid is expected to decay as a power law of index $-\frac{10}{7}$, while for compressible ideal MHD the situation is somewhat less well understood the index for the decay appears to be considerably lower than this (see e.g. Mac Low 1999 for a discussion),

As noted by Wardle & Ng (1999) and Wardle (2004), the Hall effect can be important in parts of molecular clouds where the density is in the range 10^7 – 10^{11} cm^{-3}. In this paper we use a new numerical method to perform a preliminary study of the effects on turbulence of the Hall effect and compare them with identical simulations in which ambipolar diffusion is the dominant non-ideal effect. The decay of turbulence in a single-fluid approximation incorporating the presence of the Hall effect has been studied previously by Matthaeus et al. (2003) who found no significant effect on this quantity of the turbulence.

In this paper we perform low resolution simulations of turbulent decay in a weakly ionised multifluid MHD system as a means of gaining a first insight into the behaviour of turbulence in such a system. The Hall effect, Ohmic diffusion and ambipolar diffusion are all present. We analyse the results in terms of the kinetic energy decay, the magnetic field generation/decay and the size of the structures created in the density distribution by the turbulence.

In section 2. we outline the equations used, in section 3. we give a brief overview of the numerical algorithm employed. Section 4. gives the set-up for the simulations, while we present the results themselves in section 5.. Finally, we present our conclusions in section 6..

2. The multifluid MHD equations

The equations describing multifluid MHD in the limit of low ionisation are (e.g. Falle 2003)

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{v}_i) = 0, \tag{1}$$

$$\frac{\partial \rho_1 \mathbf{v}_1}{\partial t} + \nabla \cdot (\rho_1 \mathbf{v}_1 \mathbf{v}_1 + p_1) = \mathbf{J} \times \mathbf{B}, \tag{2}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v}_1 \mathbf{B} - \mathbf{B}\mathbf{v}_1) = -\nabla \times \mathbf{E}' \tag{3}$$

$$\alpha_i \rho_i \left(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}\right) + \rho_i \rho_1 K_{i\,1} (\mathbf{v}_1 - \mathbf{v}_i) = 0, \tag{4}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{5}$$

$$\nabla \times \mathbf{B} = \mathbf{J}, \tag{6}$$

$$\sum_{i=2}^{N} \alpha_i \rho_i = 0, \qquad \sum_{i=2}^{N} \alpha_i \rho_i \mathbf{q}_i = \mathbf{J}. \tag{7}$$

The subscripts denote the species, with a subscript of 1 indicating the neutral fluid. The symbols α_i, ρ_i, $\mathbf{v}_i$ and P_i represent the charge-to-mass ratio, the mass density, velocity and pressure of species i. In the work presented here we assume an isothermal flow, although it is straightforward to include the more general energy equations in the above system. The motion of the various species are coupled through the collisional coefficients K_{ij}.

The electric field in the neutral frame, $\mathbf{E}' \equiv \mathbf{E} + \mathbf{v}_1 \times \mathbf{B}$, can be written in terms of the current, $\mathbf{J}$ as

$$\mathbf{E}' = r_{\mathrm{O}} \frac{(\mathbf{J} \cdot \mathbf{B})}{B^2} \mathbf{B} + r_{\mathrm{H}} \frac{\mathbf{J} \times \mathbf{B}}{B} + r_{\mathrm{A}} \frac{(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}{B^2} \tag{8}$$

where r_{O}, r_{H} and r_{A} are the Ohmic, Hall and ambipolar resistivities respectively. For more details see O'Sullivan & Downes (2007).

3. Numerical method

We use the numerical method described in O'Sullivan & Downes (2006; 2007). The main extra challenge in integrating the multifluid MHD equations is in dealing with the diffusion terms in the induction equation.

Where ambipolar diffusion is important (i.e. when $\eta \equiv \frac{r_{\rm A}}{r_{\rm H}}$ is large) we can use the super-time-stepping method for accelerating standard diffusion equations (e.g. Alexiades et al. 1996). It is not, however, possible to use this method for the Hall term since the eigenvalues of the Hall diffusion operator are complex. Instead we use the Hall Diffusion Scheme which allows us to integrate this term explicitly with a reasonable time-step. This means that the method is straightforward to implement in a parallel and/or adaptive mesh refinement environment. A more detailed description of this method is given in O'Sullivan & Downes (this volume).

4. Initial Conditions

We use initial conditions which are similar to those in Mac Low (1999). The computational domain is initialised as a 64^3 box of side 2. The sound speed is 0.1, so the crossing time is 20 units. The magnetic field is initially uniform in the x direction. The density is also initially uniform with $\rho = 1$ everywhere. The full parameters for the simulations are given in tables 1 and 2.

Each component of the velocity field is initialised to be a linear combination of waves with 64 wave vectors (from $\mathbf{k} = (1,1,1)^{\rm T}$ to $\mathbf{k} = (4,4,4)^{\rm T}$) each with a random amplitude and phase taken from a uniform distribution. The final velocity field is then normalised to have the appropriate root-mean-square amplitude (see table 2).

Table 1. Parameters for the ambipolar and Hall dominated simulations

Parameter	Ambipolar dominated	Hall dominated
Sound speed	0.1	0.1
ρ	1	1
α_2	-2×10^{12}	-2×10^{9}
α_3	1×10^{8}	1×10^{5}
K_{12}	4×10^{5}	4×10^{2}
K_{13}	2×10^{4}	2.5×10^{6}
ρ_2	5×10^{-8}	5×10^{-8}
ρ_3	1×10^{-6}	8×10^{-12}
η	5.86×10^{3}	4.6×10^{-3}

For each of the sets of conditions in table 2 an ambipolar and a Hall dominated simulation was run (see table 1) With the parameter ranges chosen we are exploring three different regimes: weak magnetic fields with trans- and super-Alfvénic turbulence initially, and a strong magnetic field case with trans-Alfvénic turbulence.

Table 2. Initial conditions for the simulations discussed in this work.

Simulation	**B**-field	$v_{\rm rms}$	$\beta = \dfrac{P_{\rm thermal}}{P_{\rm magnetic}}$
A	$\mathbf{B} = (0.1, 0, 0)^{\rm T}$	0.1	2
B	$\mathbf{B} = (0.1, 0, 0)^{\rm T}$	0.2	2
C	$\mathbf{B} = (0.1, 0, 0)^{\rm T}$	0.3	2
D	$\mathbf{B} = (0.1, 0, 0)^{\rm T}$	0.4	2
E	$\mathbf{B} = (0.4, 0, 0)^{\rm T}$	0.4	0.125

5. Results

We analyse the results of the simulations now in terms of the decay of the kinetic energy in the flow, the evolution of the magnetic field and the length scales of the density structures which develop in the flow. We focus on comparing results obtained in the flows which are Hall dominated with those which are ambipolar dominated.

5.1. Energy decay

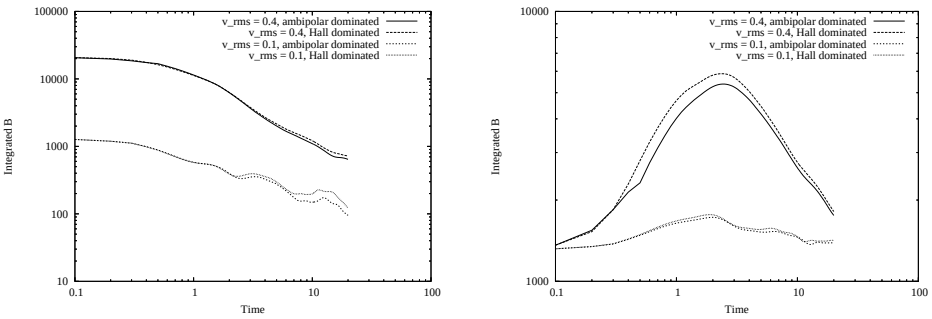

Figure 1. Evolution of the total kinetic energy (left panel) and the integrated magnetic flux for simulations A and D.

Table 3. Exponent of the kinetic energy decay with time.

Simulation	Hall dominated	Ambipolar dominated
A	0.44	0.58
B	0.68	0.76
C	0.84	0.87
D	0.93	0.98
E	0.71	0.83

Figure 1 contains plots of the decay of the total kinetic energy in the computational box as a function of time for simulations A and D. It is clear that,

after an initial transient, the decay does behave as a power law. Modelling the decay with time as $E_{\rm k} \propto t^{-\gamma}$, table 3 contains the values of γ obtained for each of the simulations. It is noticeable that, in each case the Hall dominated simulation has a lower exponent, indicating that it maintains its kinetic energy for longer than the ambipolar simulation.

While this is a systematic effect in all our simulations, the differences are generally not very large. This may in part be due to the relatively low resolution of the simulations presented in this work.

5.2. B-field evolution

Figure 1 contains plots of the integrated magnetic flux in the computational domain as a function of time for simulations A and D. It is clear that in both cases the magnetic field becomes more intense with time initially before dying away. In addition, simulation E develops more magnetic flux than simulation A. These two points can be explained as follows: as the flow evolves energy is transferred from the kinetic energy of the fluids into the magnetic field. As the kinetic energy in the motion of the fluids decays the magnetic energy is transferred back to the fluids, resulting in the decay of the magnetic field. If the initial flow is more energetic (in the case of simulation E) then we expect that more magnetic energy would be generated initially.

Finally we note that the ambipolar dominated flows do not generate quite as much magnetic field as the equivalent Hall dominated flows. This, again, is to be expected since, for significant ambipolar diffusion, the flows do not have as much "grip" on the magnetic fields so they do not generate the dynamo effect necessary to increase the magnetic energy as efficiently. Since there is no diffusion of the magnetic field in the usual sense for the Hall dominated flows, these flows can generate the magnetic energy more efficiently.

5.3. Power spectra

Table 4. Exponent of the power spectrum of density.

Simulation	Hall dominated	Ambipolar dominated
A	6.73	6.65
B	5.04	5.40
C	4.20	4.66
D	3.62	3.86
E	5.69	6.67

To gain an idea of the typical size of structure produced in each of the simulations we calculate power spectra of the density distribution in space, modelling these spectra as $P(k) \propto k^{-\gamma}$. Table 4 contains the exponents calculated for each simulation. It can be seen that, systematically, the Hall dominated simulations have harder spectra. Indeed, further analysis of the results (not shown here) show that these simulations have more power than the ambipolar dominated simulations at all scales.

One would expect that the Hall dominated flows should have more power at shorter length scales than the ambipolar dominated flows for two reasons. Firstly the Hall effect is dispersive in nature. Hence large waves will, in general, break up into short wavelength waves travelling quickly and longer wavelength waves travelling more slowly, thereby generating structure on short scales. In addition, ambipolar diffusion, unlike the Hall effect, is a truly diffusive effect. It will tend to smear out structure on short scales.

6. Conclusions

We have presented the results of simulations of turbulence in molecular clouds in the case where the flow is Hall dominated and compared these results with simulations where the flow is ambipolar dominated. For each case we have run simulations in which the initial velocity field is trans- and super-Alfvénic, and in which $\beta = 2$ and $\beta = 0.125$.

We find that the Hall effect produces more structure in the density distribution at all length scales. It allows a greater transfer of energy from the flow of the fluids to the magnetic field than is the case for the ambipolar dominated case. We also find that the kinetic energy in Hall dominated flows decays less rapidly than in equivalent ambipolar dominated flows, although this is a somewhat marginal effect in the simulations presented here, possibly due to the relatively low resolution employed.

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