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Bistability by Induced Waveguiding in Coupled Semiconductor Lasers

DANIEL M. HEFFERNAN, J. McINERNEY, L. REEKIE, AND D. J. BRADLEY

Abstract—Recently, McInerney, Reekie, and Bradley observed bistability in twin diode GaAs/GaAlAs injection lasers in an external cavity when both diodes were above threshold. We show that this bistability may be explained by a form of self-focusing which is produced by induced waveguiding in the wide stripe lasers. A detailed analysis is performed on a standard model of these diodes in an external cavity. We have found very good agreement between theory and experiment.

INTRODUCTION

Optical bistability has been observed in a wide variety of nonlinear materials including semiconductors [1], atomic vapors [2], and hybrid electro-optic systems [3]. The mechanisms used to produce bistability have relied on nonlinear absorption [4], nonlinear refraction in a resonant cavity [2], and at a nonlinear interface [5], self-focusing [6], and thermal effects [1]. Optical bistability has provided memory, optical power limiting, and differential gain [1], [7].

Recently, McInerney, Reekie, and Bradley [8] observed bistability in twin diode GaAs/GaAlAs injection lasers in an external cavity when both diodes were above threshold. We show that this bistability may be explained by a form of "self-focusing" which is produced by induced waveguiding in wide stripe lasers.

The paper is arranged as follows. We firstly outline the experiment and the results. This is followed by a detailed analysis on a model of twin diode injection lasers in an external ring cavity. A detailed comparison is then made between theory and experiment. Good agreement is found between theory and experiment. We give a detailed analysis of the self-focusing which is produced by induced waveguiding in wide stripe diodes [9] and we show that it can give rise to and explain the bistability observed by McInerney, Reekie, and Bradley.

THE EXPERIMENT

The experimental arrangement is illustrated in Fig. 1. Two GaAs/GaAlAs double heterostructure oxide isolated

stripe lasers (LD 1 and LD 2) are arranged symmetrically in a ring cavity. The cavity consists of four multilayer dielectric coated mirrors (M1-M4), each having a nominal power reflectance of 100 percent for the wavelength, polarization, and angle of incidence (45°) used. Each laser diode was nominally of length 500 μm, stripe width 20 μm, and active region thickness of 0.2 μm. Both lasers were processed from the same wafer of LPE-grown material. The overall optical length of the ring cavity was 3 m. Each facet was antireflection coated.

When the system was optimally aligned, the coupling efficiency between the active regions was of the order of 50 percent below the value required to operate the system with one diode providing gain and the other saturable loss. Light was coupled from the cavity through one of the mirrors (M1), which was shown to have a power reflectance of 99.8 percent under the conditions of use: the counterpropagating beams were sampled by two fast Si PIN photodiodes (PD 1 and PD 2, both Hewlett Packard HP 5082-4220 biased by 15 V) connected to a fast storage oscilloscope (Tektronix 7834/7A 19/7B 15).

When first bonded, diode LD 1 reached threshold at 148 mA, this value rising to 230 mA after application of antireflection coatings to both facets. For LD 2 the corresponding threshold currents were 130 mA and 171 mA, respectively. All threshold current values were taken at a heat-sink temperature of 20°C. Each antireflection coat-
EXPERIMENTAL LIGHT-CURRENT CHARACTERISTICS

The light output as a function of the current \( I_2 \) in diode LD 1 with the current \( I_2 \) in diode LD 2 held fixed is shown in Fig. 2. A similar characteristic was obtained when \( I_2 \) was varied and \( I_1 \) held fixed. The width of the hysteresis loop is of the order of 10 mA. When \( I_2 \) was increased to 190 mA, the bistability loop collapsed to a wrinkled kink (see Fig. 3). This bistable behavior was observed when both diodes were above threshold. Hence, absorptive bistability can be ruled out. Furthermore, the coupling efficiency between the diodes was of the order of 50 percent which made it impossible to observe absorptive bistability [8], [10]. Spectral bistability which has been observed in cleaved coupled cavity lasers [11] can also be ruled out as this is manifested by a jump in the lasing wavelength across many diode modes at each transition and no such jumps were observed [8]. Finally, we do not know of any refractive nonlinearity in GaAs which would be strong enough, under the conditions of this experiment, to account for the behavior of the laser.

In this paper, we show that the bistability may be explained by a form of self-focusing produced by induced waveguiding in the semiconductor lasers. In wide stripe diodes the guiding characteristics of the lasers in the junction plane depends on the gain and carrier distributions beneath the stripes [9], [12], [13]. The gain and carrier distributions are affected by the intensity of the light in the cavity, which in turn depends on the guiding characteristics of the diodes and therefore on the intensity. This intensity dependent feedback is responsible for and can explain the hysteresis loops, as is shown below. The basic mechanism can be explained as follows: When the current in one diode is held fixed, the light output intensity increases with the increasing current in the other diode. A critical intensity is reached where the lasing filament— which for optimal alignment passes through the center of each diode's active region—modifies the gain and carrier distributions beneath the stripes so as to strengthen significantly the guiding. The guiding is strengthened because of the dominance of carrier-induced guiding over gain-induced antiguiding in 20 μm oxide stripe lasers [9], [12], [13], and the effect of increasing intensity of the stimulated filament propagating through the center of the active region is to sharpen the dip in the carrier concentration under the stripe. The stronger guiding increases the intensity at the center of the stripe and, consequently, the degree of feedback and the intracavity intensity are increased. This is a runaway effect which terminates when the guiding in the diodes is such that maximum feedback is achieved, giving the upward transition of the hysteresis loop. This self-trapped condition can be maintained for lower diode currents than those required to initiate it, hence, the system is bistable and exhibits hysteresis in the light-current characteristics. A complete mathematical description of the process is given below.

THE MODEL

We will firstly examine the case of a single diode in external cavity and then extend the analysis to the double diode case. The basic equations are

\[
\frac{\partial N(x, t)}{\partial t} = \frac{J}{\varepsilon d} - \frac{N(x, t)}{\tau_s} + D \frac{\partial^2 N(x, t)}{\partial x^2} - \frac{c}{\pi} g(n(x, t)) \Gamma \Gamma I(x, t) S(t)
\]

(1)

\[
\frac{dS}{dt} = \frac{c}{\pi} \left( G - \alpha_{\text{int}} - \frac{1}{L} \ln \left( \frac{1}{R} \right) \right) S + \beta \frac{N_f}{\tau_s}
\]

(2)

The symbols are as follows. \( N \) is the carrier density. \( J \) is the current density entering the active layer. \( J \) is assumed to be constant under the stripe and zero outside the stripe region. \( d \) is the thickness of the active layer, and \( c \) is the electron charge. \( x \) is the transverse direction (to the stripe length). \( \tau_s \) is the carrier lifetime, \( n \) the group index, \( c \) the speed of light, and \( g \) is the gain per unit length. \( I(x, t) \) is the normalized intensity distribution.
\[ I(x, t) = \frac{|E(x, t)|^2}{W} \int_{-\infty}^{\infty} |E(x, t)|^2 \, dx \] (3)

where \( W \) is the stripe width. \( S \) is the photon density, \( R \) the power reflection coefficient of the end mirrors, \( L \) the length of the cavity (we assume the diode occupies the whole cavity) and \( \beta \) is the spontaneous emission factor. \( \alpha_\text{int} \) is the internal scattering loss per unit length. \( E(x, t) \) is the field in the cavity. It is the solution to

\[ \frac{\partial^2 E(x, t)}{\partial x^2} + \left( k^2 e(x, t) - \beta \right) E(x, t) = 0 \] (4)

where \( k = 2\pi/\lambda \) is the wavenumber and \( \beta \) is the propagation constant. In (1) \( \Gamma_\parallel \) is the confinement factor in the lateral direction

\[ \Gamma_\parallel = \int_{-W/2}^{W/2} |E(x, t)|^2 \, dx \int_{-\infty}^{\infty} |E(x, t)|^2 \, dx. \] (5)

\( \varepsilon(x, t) \) is the permittivity of the medium. The output power from the end mirrors is given by

\[ P = \frac{1}{2} h w \left( \frac{1}{L} \ln \left( \frac{1}{R} \right) \right) \frac{c}{n} (W \cdot L \cdot d) S(t) \] (6)

which is a product of the photon energy, mirror loss, group velocity, effective volume, and photon density. \( h \) is Planck's constant divided by \( 2\pi \). In (1) \( \Gamma_\parallel \) only appears. Mode confinement is determined primarily by lateral hole burning in the mode profile and, consequently, we have neglected confinement in the vertical direction (\( \Gamma_\perp \)) in (1). The justification for this is that it leads to good agreement with our experimental results.

The dependence of gain on carrier density is assumed to be linear

\[ g(N) = aN - b. \] (7)

The permittivity is given by

\[ \varepsilon(x, t) = n^2 + 2 \Gamma_\perp n_1 N(n, t) \frac{dn}{dn} \] (8)

where \( n \) is the effective index for the corresponding slab of thickness \( d \), and \( n_1 \) and \( n_2 \) are the refractive indexes of the active and cladding layers, respectively. The refractive index is assumed to depend linearly on carrier density, i.e., \( dn/dN = -\Delta \). The mode gain \( G \) is the average effective net gain,

\[ G(t) = \frac{1}{n} \left[ \Gamma_\perp n_1 \frac{1}{W} \int_{-\infty}^{\infty} g'(N(x, t)) I(x, t) \, dx \right] \]

\[ - (1 - \Gamma_\perp) n_2 \alpha_\text{pass} \] (9)

and the average carrier density \( N_p \) is given by

\[ N_p = \frac{1}{W} \int_{-\infty}^{\infty} N(x, t) I(x, t) \, dx \] (10)

\( g' \) represents an effective gain,

\[ g' = g(n) - (a'N - b') \] (11)

i.e., the net material gain in the active layer. \( \alpha_\text{pass} \) is the loss in the passive \( n \) and \( p \) layers.

Equations (1)–(11) have to be solved self-consistently. This is a nontrivial task, particularly since the boundary conditions for (4) must evolve self-consistently with the solution.

The above model is essentially the standard model due to Buus [14] and others [9], [12], [13], [15]–[20]. We have modified it to include more confinement in the lateral direction.

**LIGHT CURRENT CHARACTERISTICS**

The static light current characteristics can be obtained by solving (1)–(11) self-consistently. In the experimental arrangement considered above, diffusion is negligible [8]. Taking \( D = 0 \) and in the static limit, (1) gives

\[ N(x) = \frac{\tau_{\text{pass}}}{\alpha_\text{pass}} \frac{\tau_{\text{cb}}}{\alpha_\text{pass} + \alpha_\text{pass}} \Gamma_\parallel I(x) \frac{S(t)}{S(t_0)} \] (12)

We assume that the modes in the cavity are Gaussian and take

\[ I(x) = \frac{1}{W_{L}} \exp \left( -x^2/W_{L}^2 \right) \] (13)

where \( W_L \) is the full-width at half-maximum of the Gaussian mode. The physics is determined by what is happening near \( x = 0 \). Hence, for \( x << W_L \)

\[ N(x) \approx N(0) \left[ 1 + (\eta - \xi) \frac{x^2}{W_L^2} \right] \] (14)

where

\[ \xi = \frac{\tau_{\text{cb}}}{\alpha_\text{pass} + \alpha_\text{pass}} \frac{\Gamma_\parallel AS}{\Gamma_\parallel AS} \] (15)

\[ \eta = \frac{\tau_{\text{cb}}}{\alpha_\text{pass} + \alpha_\text{pass}} \frac{\Gamma_\parallel AS}{\Gamma_\parallel AS} \] (16)

and \( A = (1/\sqrt{\pi}) (W/W_L) \). \( N(0) \) is the carrier density at the center of the stripe and it is given by

\[ N(0) = \frac{\tau_{\text{pass}}}{\alpha_\text{pass} + \alpha_\text{pass}} \frac{\tau_{\text{cb}}}{\alpha_\text{pass} + \alpha_\text{pass}} \Gamma_\parallel AS \] (17)

The permittivity is thus given by

\[ \varepsilon(x) = n_0^2 \left[ 1 - 2\Delta(x^2/W_L^2) \right] \] (18)
where
\[ n_0^2 = n^2 - 2 \Gamma \Delta' \Delta N(0) \]  
and
\[ \Delta = \frac{\Gamma \Delta' N(0)}{n_0^2} (\eta - \xi) \]  
(19)
(20)

With \( \epsilon(x) \) given by (18), (4) becomes
\[ \frac{d^2 E(x)}{dx^2} + \left[ k^2 n_0^2 \left( 1 - 2\Delta \frac{x^2}{W_0^2} \right) - \beta_2^2 \right] E(x) = 0. \]  
(21)

Equation (21) has been studied in some detail in the literature (see [21] and references therein). The solutions to (21) are the Hermite-Gaussian functions [21]
\[ E_N(x) = \frac{2^{1/4}}{\pi^{1/4} N! W_0^{1/2}} H_N \left( \sqrt{2} \frac{x}{W_0} \right) \exp \left( \frac{-x^2}{2W_0^2} \right) \]  
(22)
where \( H_N \) are Hermite polynomials and the beam waist \( W_0 \) is given by [21]
\[ W_0^2 = \frac{2W_L}{kn_0 (2\Delta)^{1/2}}. \]  
(23)

The eigenmodes are given by
\[ 2N = (k^2 n_0^2 - \beta_2^2) \frac{W_0^2}{2} - 1 \]  
(24)

where \( N = 0, 1, 2, \ldots \).

We can now obtain \( I(x) \) and the use of self-consistency conditions on \( I(x) \) will give equations relating \( S \) to \( J \) and \( W_0 \) and \( \beta_2 \) to \( J, S, \) and \( W_0 \). Self-consistency on \( \epsilon(x) \) requires that
\[ W_L^2 = \frac{W_0^2}{2} \]  
and, hence, from (23)
\[ W_0 = \sqrt{\frac{k n_0 \Delta}{2}} \]  
(25)
or, equivalently, using (15), (16), and (20),
\[ \frac{\tau_J}{ed} - \frac{b}{a} = \frac{k^2 n_1 \Gamma_{\perp} \Delta'}{2 \tau_{ca}} \left( 1 + \frac{\tau_{ca}}{\pi} \Gamma_{\parallel} AS \right)^2 \]  
(26)

where
\[ A = \frac{2 W}{\sqrt{\pi W_0^2}}. \]  

Knowing \( \Gamma \) and \( W_0 \) as a function of \( J \) and \( S \), (25) would give \( S \) as a function of \( J \). The output power as a function of \( J \) may then be obtained using (6). Now \( \Gamma_{\parallel} \) may be obtained from (5)
\[ \Gamma_{\parallel} = \int_{-W_0/2}^{W_0/2} |E_N(x)|^2 dx \]  
(27)
(28)

where \( N \) denotes the \( N \)th mode. For the lowest mode
\[ \Gamma_{0\parallel} = \text{erf} \left( \frac{1}{\sqrt{2}} \frac{W}{W_0} \right) \]  
(26)
where we have used (22). Here \( \text{erf}(Z) \) is the error function.

Finally to complete the solution we need \( W_0 \) as a function of \( J \) and \( S \). This may be obtained from (2). From (10) and (14)
\[ N_p = N(0) + \frac{1}{2} \frac{k^2 n_1 \Delta'}{W_0 \Gamma_{\perp}} \]  
(27)

where \( N(0) \) is given by (17). From (9) we obtain
\[ G = \Gamma_{\perp} \frac{n_1}{n} [(a - a') N_p - (b - b')] \]  
\[ - (1 - \Gamma_{\perp}) \frac{n_2}{n} \alpha_{\text{pass}} \]  
(28)

Substituting (27) and (28) in (2) we obtain
\[ \frac{\tau_J}{ed} - \frac{b}{a} = \frac{1}{k^2 n_1 \Delta' W_0^2 \Gamma_{0\perp}} \left( 1 + \frac{\tau_{ca}}{\pi} \Gamma_{0\parallel} AS \right)^2 \]  
(29)

Equation (29) may be solved to obtain \( W_0 \) in terms of \( S \) and \( J \) and then (25) may be used to obtain \( S \) as a function of \( J \). The modes which propagate are those for which \( \beta_2 > 0 \) where \( \beta_2 \) is given by (24). As it stands this can only be done numerically. However, it represents a considerable simplification of the original equations.

The Lowest Model
The complete solution for the lowest mode is thus given by
\[ \frac{\tau_J}{ed} - \frac{b}{a} = \frac{1}{k^2 n_1 \Delta' W_0^2 \Gamma_{0\perp}} \left( 1 + \frac{\tau_{ca}}{\pi} \Gamma_{0\parallel} AS \right)^2 \]  
(30)

Knowing \( \Gamma \) and \( W_0 \) as a function of \( J \) and \( S \), (25) would give \( S \) as a function of \( J \). The output power as a function of \( J \) may then be obtained using (6). Now \( \Gamma_{\parallel} \) may be obtained from (5)
\[ \Gamma_{\parallel} = \int_{-W_0/2}^{W_0/2} |E_N(x)|^2 dx \]  
(27)
(28)

where \( N \) denotes the \( N \)th mode. For the lowest mode
\[ \Gamma_{0\parallel} = \text{erf} \left( \frac{1}{\sqrt{2}} \frac{W}{W_0} \right) \]  
(26)
where we have used (22). Here \( \text{erf}(Z) \) is the error function.

Finally to complete the solution we need \( W_0 \) as a function of \( J \) and \( S \). This may be obtained from (2). From (10) and (14)
\[ N_p = N(0) + \frac{1}{2} \frac{k^2 n_1 \Delta'}{W_0 \Gamma_{\perp}} \]  
(27)

where \( N(0) \) is given by (17). From (9) we obtain
\[ G = \Gamma_{\perp} \frac{n_1}{n} [(a - a') N_p - (b - b')] \]  
\[ - (1 - \Gamma_{\perp}) \frac{n_2}{n} \alpha_{\text{pass}} \]  
(28)

Substituting (27) and (28) in (2) we obtain
\[ \frac{\tau_J}{ed} - \frac{b}{a} = \frac{1}{k^2 n_1 \Delta' W_0^2 \Gamma_{0\perp}} \left( 1 + \frac{\tau_{ca}}{\pi} \Gamma_{0\parallel} AS \right)^2 \]  
(30)

where
\[ A = \frac{2 W}{\sqrt{\pi W_0^2}}. \]  

Knowing \( \Gamma \) and \( W_0 \) as a function of \( J \) and \( S \), (25) would give \( S \) as a function of \( J \). The output power as a function of \( J \) may then be obtained using (6). Now \( \Gamma_{\parallel} \) may be obtained from (5)
\[ \Gamma_{\parallel} = \int_{-W_0/2}^{W_0/2} |E_N(x)|^2 dx \]  
(27)
(28)
\[ \Gamma_{0\parallel} \text{ is given by (26) and the propagation constant is given by} \]
\[ \beta_{00}^{2} = k^{2} n_{0}^{2} - \frac{2}{W_{0}^{2}} \]  \hspace{1cm} (32)
\[ \text{where } n_{0} \text{ is given by (19). Equations (30)-(32) together with (6) give the power out } P \text{ as a function of } J. \]

### COUPLED SEMICONDUCTOR DIODE LASERS IN A RING CAVITY

We can extend the analysis of two semiconductor diodes in a cavity as follows. The carrier density in each diode is given by
\[ \frac{\partial N_{i}(x, t)}{\partial t} = \frac{J_{i}}{e d} - \frac{N_{i}(x, t)}{\tau_{s}} + D_{i} \frac{\partial^{2} N_{i}(x, t)}{\partial x^{2}} \]
\[ - \frac{c}{n} g_{i}(N_{i}) \Gamma_{i \parallel} I_{i}(x, t) S(t) \]  \hspace{1cm} (33)
where \( i = 1, 2 \). The photon density in the cavity is
\[ \frac{dS}{dt} = \frac{c}{n} \left[ G_{1} + G_{2} - \alpha_{i \text{int}1} - \alpha_{i \text{int}2} - \frac{1}{L} \ln \left( \frac{1}{R} \right) \right] S \]
\[ + \frac{\beta}{\tau_{s}} (N_{p1} + N_{p2}) \]  \hspace{1cm} (34)
In the above equations
\[ I_{i}(x, t) = \frac{|E_{i}(x, t)|^{2}}{1 + W_{i}^{\infty} |E_{i}(x, t)|^{2} dx} \]
\[ N_{p1} = \frac{1}{W} \int_{-\infty}^{\infty} N_{i}(x, t) I_{i}(x, t) dx \]  \hspace{1cm} (35)
and \( E_{i}(x, t) \) is obtained from
\[ \frac{d^{2} E_{i}(x, t)}{dx^{2}} + (k^{2} \epsilon_{i} + \beta_{\parallel}^{2}) E_{i}(x, t) = 0. \]  \hspace{1cm} (37)
The output power is, assuming identical diodes,
\[ P = \frac{1}{2} \hbar w \left( \frac{1}{L} \ln \frac{1}{R} \right) \frac{c}{n} (W \cdot L \cdot d) S. \]  \hspace{1cm} (38)
As before, we will assume the following:
- Linear gain: \( g_{i}(N_{i}) = aN_{i} - b \)  \hspace{1cm} (39)
- Permittivity: \( \epsilon_{i}(x, t) = n^{2} - 2\Gamma_{i \parallel} n_{1} \Delta' N_{i}(x, t) \)  \hspace{1cm} (40)
- Mode gain: \( G_{i} = \frac{1}{n} \left[ \Gamma_{i \parallel} n_{1} \frac{1}{W} \int_{-\infty}^{\infty} g_{i}^{2} I_{i}(x, t) dx \right. \]
\[ \left. - (1 - \Gamma_{i \parallel}) n_{2} \alpha_{\text{pass}} \right] \]  \hspace{1cm} (41)
\[ g_{i}' = (a - a') N_{i}(x, t) - (b - b') \]  \hspace{1cm} (42)
where \( a, a', b, b' \), etc. have the same meanings as before.

Taking the static limit and neglecting diffusion we can proceed as in the previous section to obtain
\[ C0^{2} S^{2} \left( \frac{\tau J_{1}}{ed} - \frac{b}{a} \right) = y_{1} (1 + y_{1})^{2} \]  \hspace{1cm} (43)
\[ C0^{2} S^{2} \left( \frac{\tau J_{2}}{ed} - \frac{b}{a} \right) = y_{2} (1 + y_{2})^{2} \]  \hspace{1cm} (44)
and
\[ \left( \frac{\tau J_{1}}{ed} - \frac{b}{a} \right) + \left( \frac{\tau J_{2}}{ed} - \frac{b}{a} \right) + \frac{y_{1}^{2} + y_{2}^{2}}{2C0^{2} S^{2}} = f(S) \]  \hspace{1cm} (45)
where
\[ \theta = \frac{\tau_{ca}}{n}, C = 2 k^{2} n_{1} \Delta' W^{2}/\pi, Y_{i} = \theta \sqrt{\frac{\pi W}{2 W_{0i}} S} \]
\[ f(S) = \alpha S - \frac{2}{a} \frac{b}{\tau_{s} c} \]  \hspace{1cm} (46)
and
\[ \alpha = \alpha_{i \text{int}1} + \alpha_{i \text{int}2} + \frac{2}{n} (b - b') + \frac{1}{L} \ln \left( \frac{1}{R} \right) \]
We have assumed, as is normally true physically, that one or both lasers are sufficiently above threshold so that \( \Gamma_{i \parallel} \) and \( \Gamma_{i \parallel} \neq 1 \). Finally, the lowest eigenmode \( (N = 0) \) is given by
\[ \beta_{00}^{2} = k^{2} n^{2} - 2 k^{2} n_{1} \Delta' \left[ \left( \frac{\tau J_{1}}{ed} - \frac{b}{a} \right) \right] \]
\[ \cdot (1 + Y_{i}) + \frac{b}{a} \frac{2}{W_{0i}^{2}}. \]

For fixed \( J_{2} \) we can obtain \( J_{1} \) as a function of \( S \) as follows: With \( J_{2} \) fixed, (44) gives \( Y_{2} \) as a function of \( S \)
\[ Y_{2} = \frac{1}{3} \left[ 1 + \frac{27}{2} \phi^{2} K_{2} + \frac{27}{2} \phi^{2} K_{2}^{1/3} \right]^{1/3} \]
\[ + \frac{1}{3} \left[ 1 + \frac{27}{2} \phi^{2} K_{2} - \frac{27}{2} \phi^{2} K_{2}^{1/3} \right] - \frac{2}{3} \]  \hspace{1cm} (48)
where
\[ \phi = C0S \text{ and } K_{2} = \left( \frac{\tau J_{2}}{ed} - \frac{b}{a} \right)/C. \]
Equations (43) and (45) may be coupled to give
\[ \frac{\tau J_{1}}{ed} - \frac{b}{a} = \frac{C}{27\phi^{2}} \left[ \sqrt{1 + 6F} ight. \]
\[ - 1 \left. \ast \left( \sqrt{1 + 6F} + 2 \right)^{2} \right] \]  \hspace{1cm} (49)
where,
\[ F = C0^{2} S^{2} f(S) - \frac{(3Y_{2}^{2} + 2Y_{1})}{2} \]  \hspace{1cm} (50)
Equation (49) gives \( J_1 \) as a function of \( S \) for fixed \( J_2 \). Equation (38) gives the power \( P \) as a function of \( S \). Since the current in the diodes is given by \( I_1 = W \cdot L \cdot J_1 \), (49) will give \( J_1 \) as a function of the power \( P \), for fixed \( J_2 \).

In Table I we give the parameter values for GaAs/GaAlAs. These are the standard values [14] for GaAs/GaAlAs. No attempt has been made to fit the parameters to an individual laser. It is very difficult to accurately model the losses for the experimental arrangement of Fig. 1. A gross loss is modeled by assuming an end mirror power reflectivity of 0.32. Note that this value of \( R \) comes not from facet reflectivity (they are antireflection coated) but from experimentally estimated external cavity losses. The nominal width of the injection layer was 20 \( \mu \)m. In practice the actual width was around 17 \( \mu \)m, the value we use in computing the theoretical light current characteristics. All the other parameters, i.e., \( a, a', b, b' \), etc., are the standard values for GaAs/GaAlAs [14]. The exact dependence of the real part of the refractive index on carrier density is a little uncertain. For simplicity a linear dependence given by \( dn/dN \) is assumed. The experimental results for the value of this parameter range from \( \Delta' = 4 \) to \( 8 \times 10^{-27} \) \( m^3 \) (see references [22], [23] and references therein). This range is in reasonable agreement with the theoretical estimates [24], [25]. The use of a constant value for \( \beta \) neglects the fact that \( \beta \) depends on the current in the diode and on the guiding mechanism [14], [26].

![Fig. 4. Theoretical light current characteristic with \( I_2 = 145 \) mA.](image)

![Fig. 5. Theoretical light current characteristic with \( I_2 = 180 \) mA.](image)

![Fig. 6. Theoretical light current characteristic with \( I_2 = 210 \) mA.](image)

In Figs. 4, 5, and 6 we plot the power \( P \) as a function of \( I_1 \) for fixed values of \( I_2 \). For \( I_2 \) less than 160 mA, the light current characteristic \( P - I_1 \), is just like that of an ordinary injection laser (see Fig. 4). For \( I_2 \) greater than 160 mA, we have induced waveguiding and the light current characteristic \( P - I_1 \), exhibits a kink (see Fig. 5). When \( I_2 \) is increased further, the induced waveguiding increases and the kink in the \( P - I_1 \) characteristic acquires the clas-
sics $S$ shape of static bistability (see Fig. 6). The dots and arrows indicate the actual route taken by the power with increasing and then decreasing current $I$. For the range of values we used, the condition (47) is satisfied, i.e., $\beta_{\omega I} > 0$ and the mode can propagate. The theoretical results are in good agreement with the experimental results. In theory as we increase the current $I$, we will increase the light intensity in the cavity and, hence, the bistability loop will increase in area with the increased depth of hole burning. In practice, as the intensity increases beyond a certain value we get saturation effects which lead to secondary and multiple filamentation and consequently, the bistable loop collapses to a wrinkled kink (see Fig. 3) [8]. Hence, experimentally there is a range of $I_2$ for which there is bistability. This limits the obtainable size of the bistability loops to less than 10 mA.

**Conclusions**

We have shown that induced waveguiding in coupled GaAs/GaAlAs injection lasers in a ring cavity can lead to kinking and bistability. The standard model [14]–[20] can explain quantitively the static light-current characteristics. The theory is in good agreement with the experimental results of McInerney, Reekie, and Bradley [8]. A detailed study of the switching characteristics (dynamic case) both experimentally and theoretically is now underway.

**References**


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L. Reekie, photograph and biography not available at the time of publication.

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