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Wind Turbine Power Quality Estimation Using a Lévy Model for Wind Velocity Data

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Wind Turbine Power Quality Estimation using a Lévy Model for Wind Velocity Data

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***Abstract*—The power quality of a wind turbine is determined by many factors but time-dependent variation in the wind velocity are arguably the most important. In this paper a non-Gaussian model for the wind velocity is introduced that is based on a Lévy distribution. It is shown how this distribution can be used to derive a stochastic fractional diffusion equation for the wind velocity as a function of time whose solution is characterised by the Lévy index. A numerical method for computing the Lévy index from wind velocity time series is introduced and applied to example wind velocity data for both rural and urban areas where, in the latter case, the index is observed to have a larger value. Finally, an empirical relationship is derived for the power output from a wind turbine in terms of the Lévy index using Betz law.**

I. Introduction

Developing appropriate models for assessing and predicting the *quality of power* for any renewable energy source is important throughout the energy industry. Quality of power modelling is particularly important with regard to wind energy as the construction of new wind farms is growing rapidly compared with other renewable energy systems [1]. By 2030, it is estimated that up to 40% world energy supply will be based on renewable energy sources and in countries with an appropriate disposition to generating energy from wind, wave and tidal power such as the UK and Ireland, the percentage is expected to be much higher.

Quality of power modelling is often based on a statistical analysis of the available wind velocity data which is used to assess optimum regions for the construction of wind farms [2]. Although the power generated by a wind turbine is based on a range of design factors, the wind velocity as a primary factor since, from Betz law, the power P in Watts is given by [3]

$$P = \frac{1}{2}\alpha\rho A v^3 \tag{1}$$

where v is the wind speed in metres per second (ms^{-1}), A is the area of the turbine in m^3, ρ is the density of air in kgm^{-3} and $\alpha < 0.593$ is the coefficient of performance. Although other physical factors such as air temperature and pressure, angle of attack, etc. are important, the scaling law of the output power with regard to wind velocity (i.e. $P \propto v^3$) is the most significant feature for a given design of a wind turbine with a fixed area and coefficient of performance [4]. Thus, an understanding of the time variations in the wind velocity for a given geographical location is of paramount importance with regard to locating a wind farm and monitoring its performance in terms of the power quality. This requires stochastic models to be developed for the power output [5]

The acquisition of wind velocity data over different time intervals and localities is a common practice together with a routine statistical analysis of the data. The analysis is almost exclusively based on the assumption that time variations in the wind velocity are random Brownian processes and that the rate of change of velocity as a function of time is Gaussian distributed, i.e. the wind velocity conforms to a process of diffusion. However, this is not usually the case as discussed in the following section and in this paper we develop a non-Gaussian stochastic model for the wind velocity that is based on a Lévy distribution and a fractional diffusion equation. This allows us to analyse wind velocity in terms of the Lévy index and thereby yields an approach to assessing the quality of power for a wind turbine in terms of this index. We provide examples of wind velocity data that substantiate this approach and construct an empirical relationship for the power output from a wind turbine based on the Lévy index.

II. Statistical Analysis of the Wind Speed

Figure 1 compares the wind velocity gradient $d_t v(t)$ (which represents the force generated by the wind for a unit mass computed using a forward differencing scheme) with the output from a zero-mean Gaussian distributed random number stream. The data consists 8000 samples recorded at Dublin Airport, Ireland over intervals of 1 hour from 00:00:00 on 1 January 2008 to 06:00:00 on 29 November 2008. By comparing these signals, it is clear that the statistical characteristics of $d_t v(t)$ are not Gaussian. The plot of $d_v t$ obtained from the wind velocity data clearly shows that there are a number of rare but extreme events corresponding to short periods of time over which the change in wind velocity is relatively high. This leads to a distribution with a narrow width but longer tail when compared to a normal (Gaussian) distribution. Non-Gaussian distributions of this type are typical of Lévy processes which are discussed in the following section.

III. Lévy Processes

Lévy processes are random walks whose distribution has infinite moments. The statistics of (conventional) physical

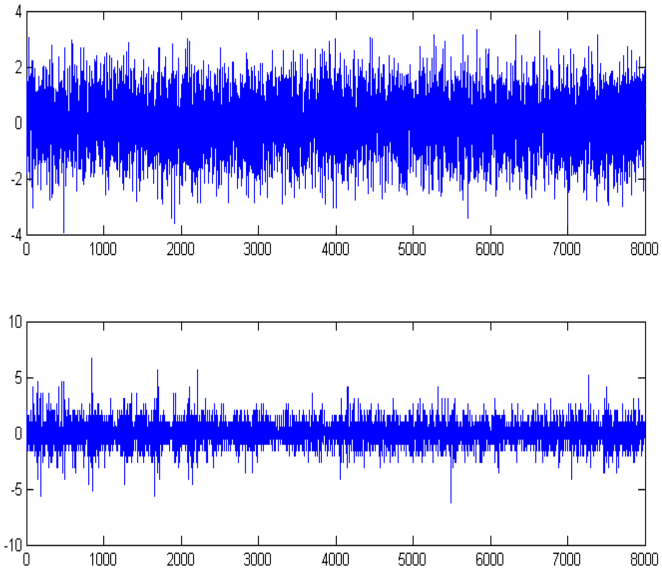

Fig. 1. Plots of a zero-mean Gaussian distributed stochastic signal obtained using MATLAB V7 *randn* function (above) and the gradient of the wind velocity (below).

systems are usually concerned with stochastic fields that have PDFs (Probability Density Functions) where (at least) the first two moments (the mean and variance) are well defined and finite. Lévy statistics is concerned with stochastic processes where all the moments (starting with the mean) are infinite. Many distributions exist where the mean and variance are finite but are not representative of the process, e.g. the tail of the distribution is significant, where rare but extreme events occur. These distributions include Lévy distributions [6]. Lévy's original approach to deriving such distributions is based on the following question: Under what circumstances does the distribution associated with a random walk of a few steps look the same as the distribution after many steps (except for scaling)? This question is effectively the same as asking under what circumstances do we obtain a random walk that is statistically self-affine. The characteristic function $P(k)$ of such a distribution $p(x)$ was first shown by Lévy to be given by (for symmetric distributions only) [6]

$$P(k) = \exp(-a \mid k \mid^{\gamma}), \quad 0 < \gamma \leq 2 \tag{2}$$

where a is a constant and γ is the Lévy index. For $\gamma \geq 2$, the second moment of the Lévy distribution exists and the sums of large numbers of independent trials are Gaussian distributed. If a stochastic process is characterised by a random walk with a step length distribution governed by $p(x)$ with $\gamma = 2$, then the result is normal (Gaussian) diffusion, i.e. a Brownian random walk process. For $\gamma < 2$ the second moment of this PDF (the mean square), diverges and the characteristic scale of the walk is lost. For values of γ between 0 and 2, Lévy's characteristic function corresponds to a PDF of the form

$$p(x) \sim \frac{1}{x^{1+\gamma}}, \quad x \to \infty$$

Furthermore, Lévy processes characterised by a PDF of this type conform to a fractional diffusion equation as we shall now show [7].

The evolution equation for random walk processes that generating a macroscopic field denoted by $f(x,t)$ is given by

$$f(x,t+\tau) = f(x,t) \otimes_x p(x)$$

where $\otimes_x$ denotes the convolution integral over x and $p(x)$ is an arbitrary PDF. We consider the field f to be the force generated by the wind velocity v which for unit mass is given by $f = \partial_t v$. From the convolution theorem, in Fourier space, this equation becomes

$$F(k,t+\tau) = F(k,t)P(k)$$

where F and P are the Fourier transforms of f and p respectively. From equation (2), we note that

$$P(k) = 1 - a \mid k \mid^{\gamma}, \quad a \to 0$$

so that we can write

$$\frac{F(k,t+\tau) - F(k,t)}{\tau} \simeq -\frac{a}{\tau} \mid k \mid^{\gamma} F(k,t)$$

which for $\tau \to 0$ gives the fractional diffusion equation

$$\sigma \frac{\partial}{\partial t} f(x,t) = \frac{\partial^{\gamma}}{\partial x^{\gamma}} f(x,t), \quad \gamma \in (0,2]$$

where $\sigma = \tau / a$ and we have used the result

$$\frac{\partial^{\gamma}}{\partial x^{\gamma}} f(x,t) = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \mid k \mid^{\gamma} F(k,t) \exp(ikx) dk$$

We note that the same fractional diffusion equation can be considered for the wind velocity since $f = \partial_t v$ giving

$$\sigma \frac{\partial}{\partial t} v(x,t) = \frac{\partial^{\gamma}}{\partial x^{\gamma}} v(x,t), \quad \gamma \in (0,2] \tag{3}$$

This derivation of the fractional diffusion equation reveals its physical origin in terms of the non-Gaussian statistics associated with the gradient of the wind velocity as illustrated Figure 1 modelled using a Lévy distribution for $a \to 0$.

For normalized units $\sigma = 1$ we consider equation (3) for a 'white noise' source function $n(t)$ and a spatial impulse function $-\delta(x)$ so that

$$\frac{\partial^{\gamma}}{\partial x^{\gamma}} v(x,t) - \frac{\partial}{\partial t} v(x,t) = -\delta(x) n(t), \quad \gamma \in (0,2]$$

which, ignoring (complex) scaling constants, has the Green's function solution [8]

$$v(t) = \frac{1}{t^{1-1/\gamma}} \otimes_t n(t) \tag{4}$$

where $\otimes_t$ denotes the convolution integral over t and $v(t) \equiv v(0,t)$. The function $v(t)$ has a Power Spectral Density Function (PSDF) given by (for scaling constant c)

$$\mid V(\omega) \mid^2 = \frac{c}{\mid \omega \mid^{2/\gamma}}$$

where

$$V(\omega)=\int_{-\infty}^{\infty} v(t)\exp(-i\omega t)dt$$

and a self-affine scaling relationship

$$\Pr[v(at)]=a^{1/\gamma}\Pr[v(t)]$$

for scaling parameter $a>0$ where $\Pr[v(t)]$ denotes the PDF of $v(t)$. This scaling relationship means that the statistical characteristics of $v(t)$ are invariant of time accept for scaling factor $a^{1/\gamma}$. Thus, if $v(t)$ is taken to be the wind velocity as a function of time, then the statistical distribution of this function will be the same over different time scales whether, in practice, it is sampled in hours or seconds, for example.

IV. Lévy Index Analysis

The PSDF $|\ V(\omega)\ |^2$ provides a method of computing γ using the least squares method based on minimizing the error function

$$e(c,\gamma)=\|2\ln\mid V(\omega)\mid-\ln c-2\gamma^{-1}\ln\mid\omega\mid\|_2^2,\quad\omega>0$$

Figures 2 shows the computation of $\gamma(t)$ for a moving window of size 1024 elements. The accompanying table (Table I provides some basic statistical information with regard to $\gamma(t)$ for these data sets. Application of the Bera-Jarque parametric hypothesis test of composite normality is rejected (i.e. 'Composite Normality' is of type 'Reject') and thus $\gamma(t)$ is not normally distributed.

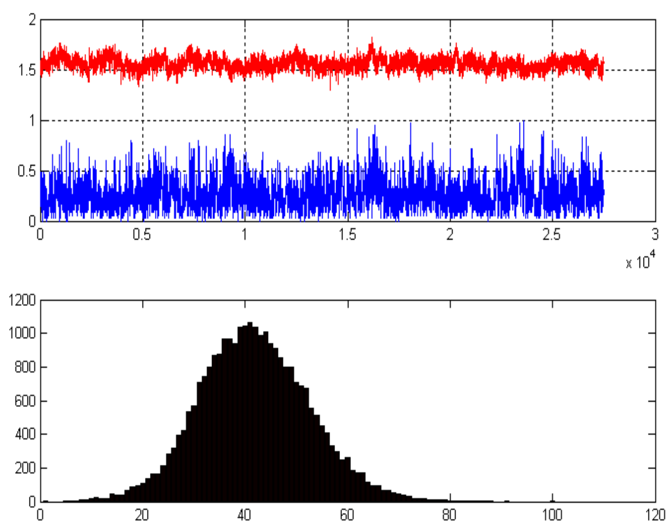

Fig. 2. Cork Airport (12/11/2003-1/1/2007) for hourly (averaged) sampled data. Above: Normalised wind velocity data $v(t)$ (blue) and the Lévy index $\gamma(t)$ (red) for a look-back moving window of 1024 elements. Below: 100-bin histogram of $\gamma(t)$.

These result illustrates that the wind velocity function is a self-affine stochastic function with a mean Lévy index of ~ 1.5. Based on these results, Figure 3 shows a simulation of the wind velocity based on the computation of $v(t)$ in equation (4) for $\gamma=1.5$. The simulation is based on transforming equation (4) into Fourier space and using a Discrete Fourier Transform. The function $n(t)$ is computed using MATLAB (V7) uniform random number generator *rand* for $seed=1$.

TABLE I
Statistical parameters associated with the Lévy index function given in Figure 2.

Statistical Parameter	Value for $\gamma(t)$
Minimum Value	1.3001
Maximum value	1.8142
Range	0.5141
Mean	1.5615
Median	1.5613
Standard Deviation	0.0569
Variance	0.0032
Skewness	0.0759
Kertosis	3.1966
Composite Normality	'Reject'

The results given in Figure 2 are for wind velocity data obtained a rural areas. It is interesting to note that, in urban areas, the Lévy index may be expected to increase as a result of the further 'diffusion' of the wind velocity through 'random scattering' of the wind from buildings in the local vicinity when, according the model being considered, $\gamma\rightarrow 2$. An example of this is given in Figure 4 and Table II in which the average Lévy index is ~ 1.72 thereby confirming this expectation.

V. Power Quality Estimation for Wind Energy Generation

Given equation (1) and equation (4), we can obtain an expression for the power output by a wind turbine in terms of the Lévy index γ as a function of time. Let the noise function in equation (4) be a simple impulse at an instant in time so that $n(t)=\delta(t)$. Then

$$v(t)=\frac{1}{t^{1-1/\gamma}}$$

and, from equation (1),

$$P(t)=\frac{\beta}{t^{3(1-1/\gamma)}}$$

where $\beta=\alpha\rho A/2$ so that

$$\ln P(t)=\ln\beta-3\ln t+\frac{3}{\gamma}\ln t$$

Given that β is a constant, it is then clear that, for any time t, the magnitude of $\ln P$ is determined by γ^{-1}. In this sense, γ^{-1} is a coefficient of power quality as a function of time and we see that, according to this model, power output increases as γ decreases. Thus, the signal $\gamma(t)$ given in Figure 2, for example, represents a time varying measure of the average output power at a time τ according to the scaling law

$$\langle\ln P(t)\rangle_\tau=A+\frac{B}{\gamma(\tau)}$$

where $\langle\ln P(t)\rangle_\tau$ denotes the (moving) average value of $\ln P(t)$ at a time τ and A and B are scaling constants associated with a given wind turbine obtained by calibration.

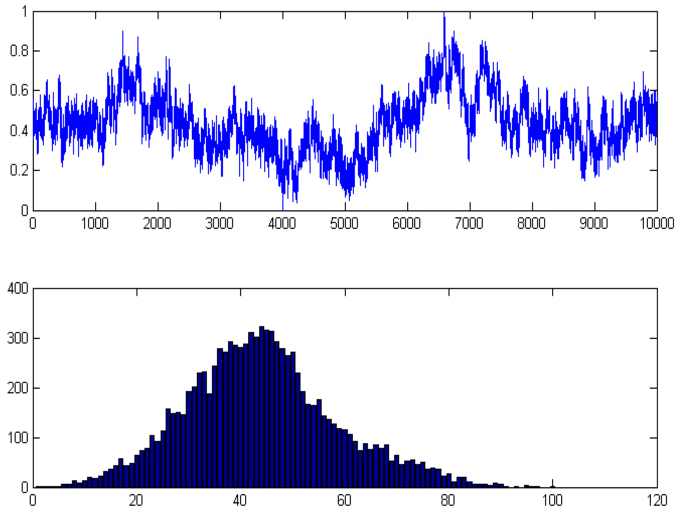

Fig. 3. Simulated normalised wind velocities computed for a Lévy index $\gamma = 1.5$ (above) and the corresponding 100-bine histogram (below)

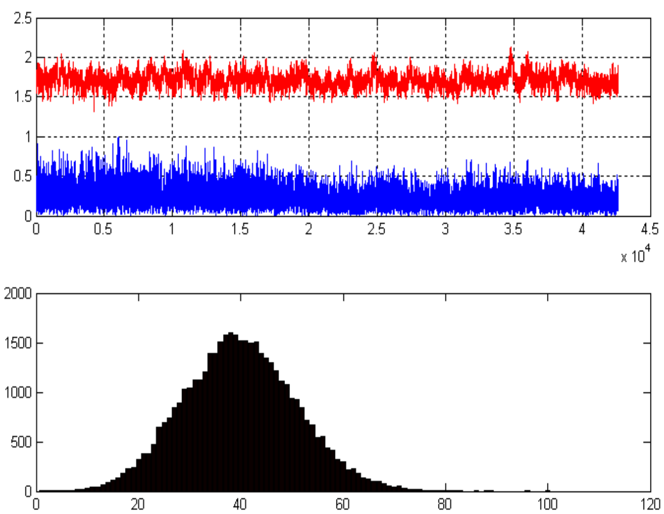

Fig. 4. Example of urban data analysis using wind velocities recorded at Dublin Institute of Technology, Kevin Street, Dublin 8 from 14 September 2010 at 22:20:44 to 15 September 2010 at 10:11:51 and sampled in seconds. Above: Normalised wind velocity data $v(t)$ (blue) and the Lévy index $\gamma(t)$ (red) for a look-back moving window of 1024 elements. Below: 100-bin histogram of $\gamma(t)$.

TABLE II
STATISTICAL PARAMETERS ASSOCIATED WITH THE LÉVY INDEX FUNCTION GIVEN IN FIGURE 4.

Statistical Parameter	Value for $\gamma(t)$
Minimum Value	1.3209
Maximum value	2.1358
Range	0.8149
Mean	1.7236
Median	1.7204
Standard Deviation	0.0944
Variance	0.0089
Skewness	0.1939
Kertosis	3.0374
Composite Normality	'Reject'

VI. SUMMARY

We have considered a Lévy distributed model and constructed a fractional diffusion equation for the wind velocity whose temporal solution is characterised by the Lévy index. Analysis of wind velocity data (some examples of which have been provided in this paper) according to this model shows that the Lévy index is a time varying non-Gaussian stochastic function. Based on the data analysed to date, the index appears to be larger ~ 1.7 for urban areas compared to rural areas when $\gamma \sim 1.5$. These results are consistent with the underlying rationale associated with the model, where, as $\gamma \rightarrow 2$, the stochastic processes become increasingly diffusive. The model presented allows times series for wind velocity to be simulated whose statistical properties are consistent with experimental data (e.g. Figure 3. Moreover, based on the calculations performed in Sections V, the Lévy index may provide a useful measure on the power quality of wind turbines. Further investigation are required to ascertain whether it may be possible to use the signal $\gamma(t)$ for short term predictive analysis on power quality following methods developed for financial risk management [9].

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