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# System identification using higher order spectra

Aidan O'Dwyer
*Dublin Institute of Technology*, aidan.odwyer@dit.ie

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Aidan O'Dwyer,
School of Control Systems and
Electrical Engineering,
D.I.T. Kevin St., Dublin 8.
E-mail: aidan.odwyer@dit.ie

## Introduction

1. This communication will explore the use of particular types of frequency domain concepts, labelled ***higher order spectra***, to help determine the ***frequency response*** of a process. This will allow the identification of the parameters of a single input, single output (SISO) model for a process (which includes a time delay), in both open loop and closed loop environments.

2. Process models with time delays arise in many signal processing applications, such as in underwater tracking applications, biomedicine, geophysics, astronomy, acoustics, seismology and telecommunications.

3. The estimation of accurate models in these applications is bedeviled by, for example, the presence of ***additive, coloured Gaussian noise*** that is often present on the signals. The use of higher order spectra in system identification allows the ***suppression*** of such noise; it also allows the recovery of phase information from signals (which cannot be done using power spectral density techniques) and allows the detection and quantification of nonlinearities.

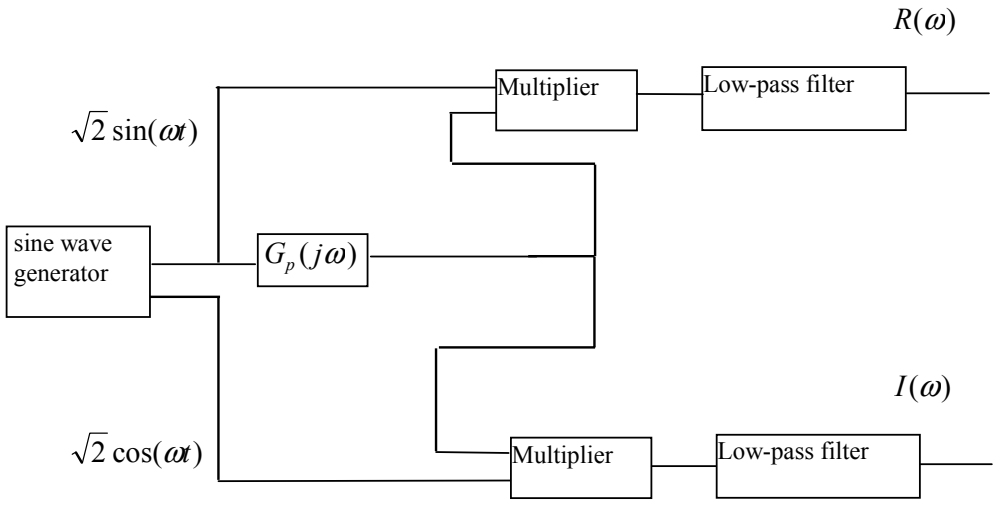
4. The communication will, in addition, explore the ***literature*** associated with the use of higher order spectra, will detail research results and will recommend the conditions in which the use of higher order spectral techniques are indicated.

# 1. Conventional frequency domain techniques

The frequency response of a process (in open loop) at any frequency may be found by calculating the gain and phase of the process, from its output, when an appropriate ***sine wave*** is input to the process. However, the estimate obtained is sensitive to disturbances.

Alternatively, the frequency response of a process may be found by finding the response of the process, in open loop, to a ***pulse*** input.

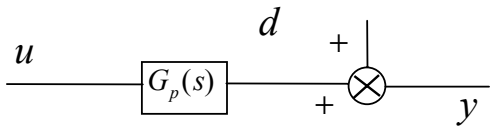
The frequency response may be found directly by ***correlation***. This approach may be represented as follows:




The process frequency response, $G(j\omega)$, equals $R(\omega) + jI(\omega)$. The method is insensitive to step and white noise disturbances. However, ***long experiment times*** are often required to determine the process frequency response.

# 2. Fourier transform ratios/power spectral density

In **open loop**, the system considered is represented as follows:




with u(t) and d(t) being uncorrelated, d(t) is a general disturbance term. The plant frequency response is estimated as

$$G_p(j\omega) = \frac{Y(j\omega)}{N(j\omega)}$$

with $N(j\omega)$ and $Y(j\omega)$ being the ***Fourier transforms*** of n(t) and y(t).

Alternatively, an estimate of the magnitude of the frequency response of the process, $\left|\hat{G}_p(j\omega)\right|$, may be determined as follows:

$$\left|\hat{G}_p(j\omega)\right| \approx S_{yu}(j\omega)/S_u(j\omega)$$

with $S_{yu}(j\omega)$ equal to the power spectral density of y(t) with respect to u(t) and $S_u(j\omega)$ equal to the power spectral density of u(t). The power spectral densities may be estimated in two separate ways:

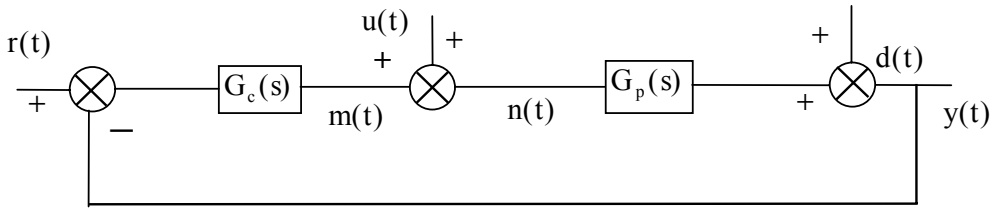
1. ***Periodogram*** approach: This method involves estimating the power spectral density in terms of the square of the corresponding discrete Fourier transform.
2. ***Correllelogram*** approach: This method involves estimating the relevant covariance functions, and calculating the estimates of the power spectral densities from the discrete Fourier transforms of these covariance functions

Phase information is lost if the power spectral density method is used. However, Schwartzenbach and Gill [1] declare that

$$e^{-2j\phi} = S_{uy}(j\omega)/S_{yu}(j\omega)$$

with $\phi =$ phase of $\hat{G}_p(j\omega)$.

The **closed loop** system considered may be represented as follows:




If r(t), u(t) and d(t) are uncorrelated, then it may be demonstrated that [2]

$$\frac{F[y(t)]}{F[u(t)]} = \frac{F[d(t)] + G_c(j\omega)G_p(j\omega)F[r(t)] + G_p(j\omega)F[u(t)]}{-G_c(j\omega)F[d(t)] + G_c(j\omega)F[r(t)] + F[u(t)]}$$

Then, in the special case that F[d(t)] = 0,

$$G_p(j\omega) = F[y(t)]/F[n(t)]$$

Alternatively, if r(t) = d(t) = 0, then it may be proven that [2]

$$G_p(j\omega) = S_{my}(j\omega)/S_m(j\omega)$$

It can be shown that, under the same circumstances [3],

$$G_p(j\omega) = S_{ny}(j\omega)/S_n(j\omega)$$

If r(t), u(t) and d(t) are uncorrelated, then it may be demonstrated that [3]

$$G_p(j\omega) = S_{uy}(j\omega)/S_{un}(j\omega)$$

or

$$G_p(j\omega) = S_{ry}(j\omega)/S_{rn}(j\omega)$$

# 3. Higher order spectral techniques

Higher order spectra (or polyspectra) are defined in terms of the higher order statistics (or cumulants) of a signal. The particular higher order spectra of most interest are

- the third order spectrum (also called the ***bispectrum***) and
- the fourth order spectrum (also called the ***trispectrum***).

The power spectrum (or power spectral density) is a second order spectrum.

The general motivations for the use of higher order spectral techniques are [4]

- to suppress additive, possibly coloured Gaussian noise that may be present on signals
- to allow recovery of phase information from signals and
- to detect and quantify nonlinearities in time series.

**Mathematical definitions**

The most common higher order spectra of a signal that are calculated are the **bispectrum** and **trispectrum**, as defined below.

Bispectrum:

- For continuous time signals,

$$B(\omega_1,\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_3(\tau_1,\tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)}d\tau_1 d\tau_2$$

with $c_3(\tau_1,\tau_2)$ being the third order ***moment*** or ***cumulant*** signal, defined as

$$c_3(\tau_1,\tau_2) = E\{X(t)X(t+\tau_1)X(t+\tau_2)\}$$

and with X(t) being a real, stationary stochastic or deterministic signal.

- For discrete time signals,

$$B(\omega_1,\omega_2) = \sum_{m_1=-\infty}^{\infty}\sum_{m_2=-\infty}^{\infty} c_3(m_1,m_2)e^{-j(\omega_1 m_1+\omega_2 m_2)}$$

with $c_3(m_1,m_2)$ being the third order cumulant sequence, defined as

$$c_3(m_1, m_2) = E\{X(n)X(n+m_1)X(n+m_2)\}$$

and with X(n) being samples of a real, stationary stochastic or deterministic sequence.

Trispectrum:

- For continuous time signals,

$$T(\omega_1, \omega_2, \omega_3) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c_4(\tau_1, \tau_2, \tau_3) e^{-j(\omega_1\tau_1 + \omega_2\tau_2 + \omega_3\tau_3)} d\tau_1 d\tau_2 d\tau_3$$

with $c_4(\tau_1, \tau_2, \tau_3)$ being the fourth order cumulant signal, defined as

$$c_4(\tau_1, \tau_2, \tau_3) = E\{X(t)X(t+\tau_1)X(t+\tau_2)X(t+\tau_3)\} - c_2(\tau_1)c_2(\tau_3 - \tau_2) - c_2(\tau_2)c_2(\tau_3 - \tau_1) - c_2(\tau_3)c_2(\tau_2 - \tau_1)$$

and with $c_2(\tau)$ being the autocorrelation function and X(t) being a real, stationary stochastic or deterministic signal.

- For discrete time signals

$$T(\omega_1, \omega_2, \omega_3) = \sum_{m_1=-\infty}^{\infty}\sum_{m_2=-\infty}^{\infty}\sum_{m_3=-\infty}^{\infty} c_4(m_1, m_2, m_3) e^{-j(\omega_1 m_1 + \omega_2 m_2 + \omega_3 m_3)}$$

with $c_4(m_1, m_2, m_3)$ being the fourth order cumulant sequence, defined by

$$c_4(m_1, m_2, m_3) = E\{X(n)X(n+m_1)X(n+m_2)X(n+m_3)\} - c_2(m_1)c_2(m_3 - m_2) - c_2(m_2)c_2(m_3 - m_1) - c_2(m_3)c_2(m_2 - m_1)$$

and $c_2(m)$ being equal to the autocorrelation sequence and X(n) being a real, stationary stochastic or deterministic sequence.

Cross-cumalents and the cross-bispectrum or cross-trispectrum may be defined in a similar manner. The bispectrum and trispectrum are special cases of the $n^{th}$ order spectrum of a signal. Generally speaking, for computational purposes, the bispectrum of a signal is the most often calculated; the trispectrum of the signal may be calculated if the signal had zero (or very small) third order cumulants and larger fourth order cumulants. A symmetrically distributed random variable has a third order cumulant equal to zero, for instance [5].

The ***cepstrum*** of higher order spectra may also be defined, as follows:

Bicepstrum: (or cepstrum of the bispectrum)

For sampled signals,

$$b(m,n) = Z_2^{-1}\{\ln[B(z_1,z_2)]\}$$

with $z_1 = e^{j\omega_1}$, $z_2 = e^{j\omega_2}$ and $Z_2^{-1}\{\}$ being the two dimensional inverse Z transform of the function. The bicepstrum may be shown to be equal to

$$b(m,n) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \ln[B(\omega_1,\omega_2)]e^{j(\omega_1 m+\omega_2 n)}d\omega_1 d\omega_2$$

Tricepstrum: (or cepstrum of the trispectrum)

For sampled signals,

$$t(m,n,l) = Z_3^{-1}\{\ln[T(z_1,z_2,z_3)]\}$$

with $z_1 = e^{j\omega_1}$, $z_2 = e^{j\omega_2}$, $z_3 = e^{j\omega_3}$ and $Z_3^{-1}\{\}$ being the three dimensional inverse Z transform of the function. The tricepstrum may be shown to be equal to

$$t(m,n,l) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \ln[T(\omega_1,\omega_2,\omega_3)]e^{j(\omega_1 m+\omega_2 n+\omega_3 l)}d\omega_1 d\omega_2 d\omega_3$$

**Parameter estimation techniques using higher order spectra**

1. An important means of finding the parameters of the process model in the frequency domain is to first find the magnitude and phase variation of the process model with frequency. An intermediate stage may be to find the ***bispectrum or trispectrum magnitude and phase estimates*** of the process [4]. The ***process magnitude and phase*** may be estimated from the bispectral magnitude and phase estimates of the process found [4], [6-12].

2. It is also possible to find *the* ***bicepstrum and tricepstrum*** of the input and output data, as an intermediate stage to ***finding the process magnitude and phase*** estimates [13-15].

3. The ***direct estimation of the process model parameters using higher order spectral techniques*** (without first estimating the process magnitude and phase) has been addressed, under the following topic headings:
   - the estimation of the parameters of an Auto Regressive Moving Average (ARMA) process model, in the z domain [4, 16-21]
   - the estimation of the most appropriate order of the numerator and denominator polynomials in the ARMA process model [17, 18].
   - the estimation of both the process parameters and time delay, in the discrete time domain [22].

4. The ***direct estimation of the time delay between two signals using higher order spectral techniques*** (without first estimating the process magnitude and phase). No other process dynamics are considered. These methods are divided into the following categories [4]:
   - Conventional time delay estimation techniques based on third order statistics. These methods involve maximising the integral of a function that depends on the bispectral and cross-bispectral phases of the input and output signals to the process [17, 23-31].
   - Parametric time delay estimation techniques in the bispectral or trispectral domain. These methods involve modelling the time delay by a polynomial and estimating the polynomial coefficients [20, 26, 32].
   - Time delay estimation techniques based on the cross-bicepstrum [33, 34].
   - Time delay estimation techniques based on the mean fourth-cumulant criterion. This method is based on the trispectral domain.
   - Adaptive time delay estimation based on the parametric modelling of higher order cross-cumulants [35, 36].

**One estimation technique**

If the signal x(k) has a Fourier transform $X(\omega)$, then the ***bispectral phase*** $\Psi_x(\omega_1,\omega_2)$ and the ***process phase*** $\phi_x(\omega)$ are related by

$$\Psi_x(\omega_1,\omega_2) = \phi_x(\omega_1) + \phi_x(\omega_2) - \phi_x(\omega_1+\omega_2)$$

It may be deduced that the bispectral phase is ***blind*** to the presence of time delay terms (as $\phi_x(\omega) = -\omega\tau, \tau =$ time delay), and thus the process phase recovered from the bispectral phase will not include a contribution from the time delay term.

The ***bispectral magnitude*** $|B_x(\omega_1,\omega_2)|$ and the ***process magnitude*** $|X(\omega)|$ are related by

$$|B_x(\omega_1,\omega_2)| = |X(\omega_1)||X(\omega_2)||X(\omega_1+\omega_2)|$$

A method of finding the process phase that ***does include a contribution*** from the time delay term is to find the cross-bispectrum of the input and output signals to a process with time delay. If n(k) is the input signal to the process and y(k) is the output signal with corresponding Fourier transforms $X(\omega)$ and $Y(\omega)$, then

$$\Psi_{xyx}(\omega_1,\omega_2) = \phi_x(\omega_1) + \phi_y(\omega_2) - \phi_x(\omega_1+\omega_2)$$

and

$$\Psi_x(\omega_1,\omega_2) = \phi_x(\omega_1) + \phi_x(\omega_2) - \phi_x(\omega_1+\omega_2)$$

Then $\phi_y(\omega_2) - \phi_x(\omega_2)$ (process phase) = $\Psi_{xyx}(\omega_1,\omega_2) - \Psi_x(\omega_1,\omega_2)$. Also,

$$|B_{xyx}(\omega_1,\omega_2)| = |X(\omega_1)||Y(\omega_2)||X(\omega_1+\omega_2)|$$

and

$$|B_x(\omega_1,\omega_2)| = |X(\omega_1)||X(\omega_2)||X(\omega_1+\omega_2)|$$

Therefore, $\dfrac{|B_{xyx}(\omega_1,\omega_2)|}{|B_x(\omega_1,\omega_2)|} = \dfrac{|Y(\omega_2)|}{|X(\omega_2)|}$ = process magnitude at frequency $\omega_2$.

The process magnitude and phase may also be obtained in terms of other cross-bispectra of the input and output signals, as well as in terms of a number of cross-trispectra of the input and output signals.

**Implementation issues**

1. Accuracy of estimates obtained: The mean and standard deviation of the process magnitude and phase estimates found using a number of higher order spectral approaches, from a finite set of original data, depends on
   - The method used to determine the bispectral magnitude and phase estimates [4]
   - The number of data points used, and whether they are overlapping [29].

   One author [10] shows in simulation that for 2048 points of data, the range of values of process phase estimated may vary $\pm$ ***17%*** around the nominal value of process phase and the range of values of process magnitude estimated may vary $\pm$ ***64%*** around the nominal process magnitude value

2. Computational intensity of the methods used: Higher order spectral techniques are more computationally intensive than are power spectral density approaches. ***A simplified view*** of this issue is that since the bispectral parameter estimate of the process input and output (for example) involves the ratio of the product of three FFT terms in their implementation, it will be more computationally intensive than a power spectrum approach (involving the ratio of the product of two FFT terms) or an approach that estimates the transfer function as the ratio of the FFT of the output of the process to the FFT of the input of the process.
   - It is possible to cut down on the computational intensity of finding the bispectral parameter estimates by the use of an appropriate algorithm.
   - However, the number of data points needed for a reliable estimate of the process gain and phase using the bispectral approach is often greater than the number of data points required for the process gain estimate using the power spectral approach [5, 20].

3. Robustness of the estimates using higher order spectra:

   **The higher order spectra of Gaussian signals is identically zero**. Thus, if additive Gaussian noise of unknown spectrum is added to the process input or output, the estimation should not be affected [16, 17, 19, 20]. Successful identification of the process using higher order spectral methods requires
   - the **input signal to the process is a non-Gaussian, zero-mean, independent and identically distributed (i.i.d) random variable** with finite moment values filtered through a finite dimensional, asymptotically stable linear transfer function
   - the **noise signals on the input and output of the process are i.i.d, possible mutually correlated, coloured Gaussian** or non-Gaussian random variables (with a symmetric probability density function, if non-Gaussian). In many practical situations, noise fulfils this criteria [19].

**Simulation result – bispectral method**

A simulation result that illustrates the estimation of the process magnitude and phase from the bispectral magnitude and phase is provided in Figures 1 to 4. The simulated process transfer function is

$$G_p(s) = 1/(s+1)$$

A Hamming window is used. The process input is assumed to be a pseudo-random binary signal (PRBS). Figures 1 and 2 illustrate the bispectrum magnitude and phase determined, with Figures 3 and 4 showing the subsequent process magnitude and phase estimates calculated, using the Lii-Rosenblatt procedure ([4], p. 322); the first magnitude and phase values are initialised [7].

Figure 1: Bispectrum magnitude
(PRBS input, 64 points, Hamming window)

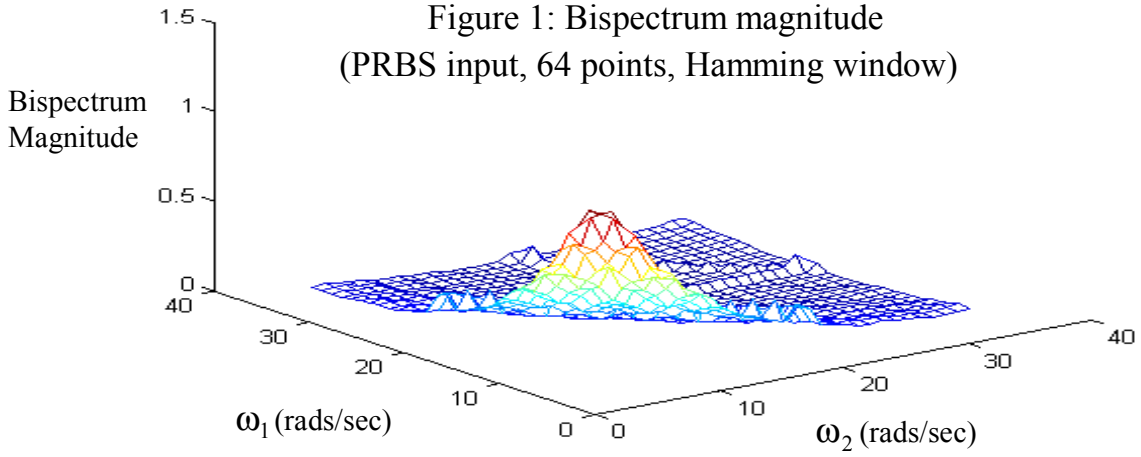



Figure 2: Bispectrum phase
(PRBS input, 64 points, Hamming window)

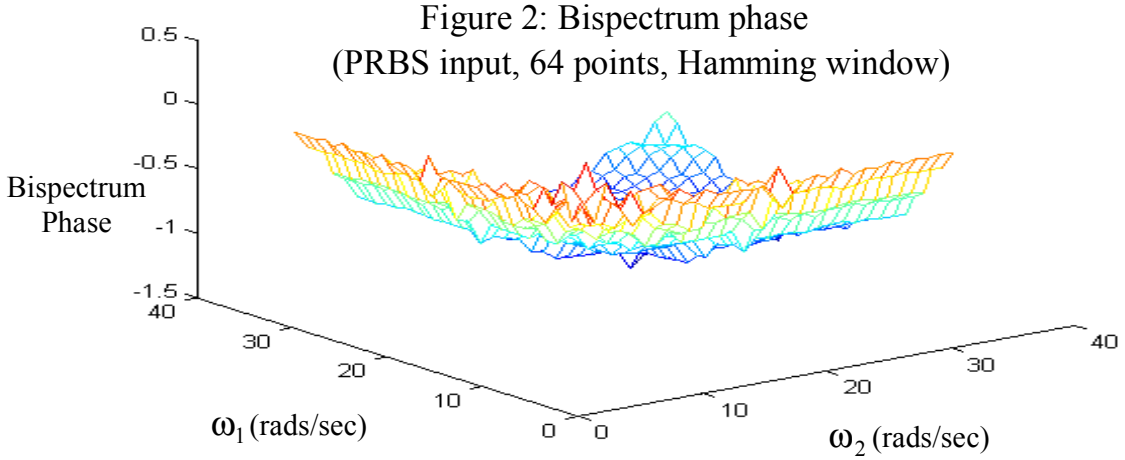


Figure 3: Simulated process and model magnitude
(PRBS input, 64 points, Hamming window)

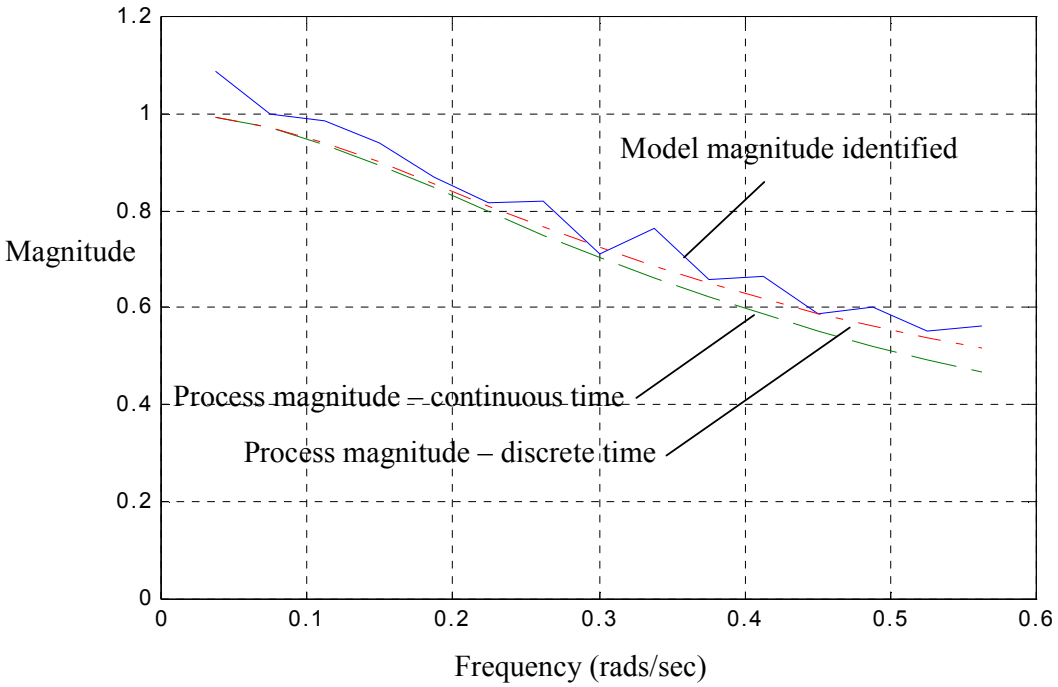
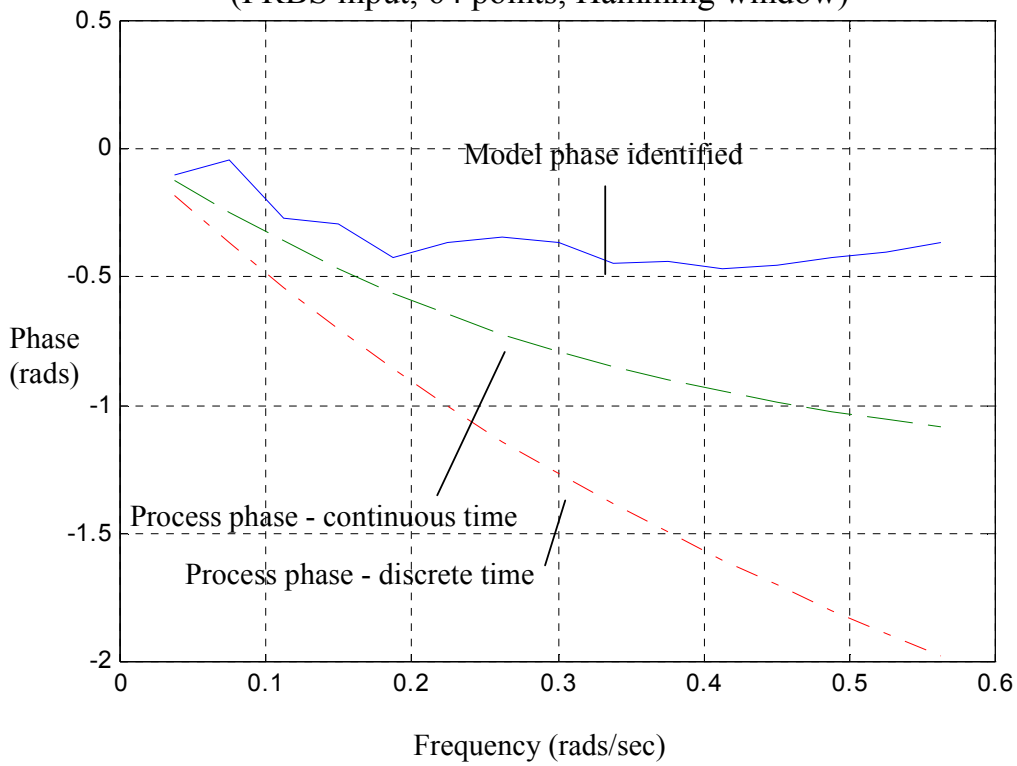



Figure 4: Simulated process and model phase
(PRBS input, 64 points, Hamming window)



1. The full panorama of results show that the quality of the process magnitude and (in particular) the process phase estimates obtained from the bispectral magnitude and phase estimates ***increases with the number of data points taken***; the choice of window taken in the implementation had less of an influence on the estimates of the process magnitude and phase estimates.

2. However, taking the full panorama of results, the process magnitude estimation using higher order spectra appears to be little improved over the power spectral density approach. The ***use of higher order spectral techniques*** instead of the power spectral density approach ***for process magnitude estimation*** could only be justified ***if the level of additive Gaussian noise on the process was large***, outweighing the computational disadvantage (and possibly, the large standard deviation associated with the process gain estimate) of the method.

3. ***One further possibility*** for process parameter estimation would be to use the higher order spectral approaches to estimate the non-time delay terms, and to use an alternative method to find the process phase (which includes the effect of a time delay term); the time delay may then be deduced by subtraction of this process phase estimate from the process phase estimate found using the higher order spectral approaches.

**Simulation result – cross-bispectral method – open loop**

A simulation result that illustrates the estimation of the process magnitude and phase from the cross-bispectrum of the input and output signals is provided in Figures 5 and 6. The simulated process transfer function taken is

$$G_p(s) = e^{-1.66s} / (s+1)$$

A Hamming window is used in the simulations. The process input is assumed to be in PRBS form.

Figure 5: Simulated process and model magnitude
(PRBS input, 128 points, Hamming window)

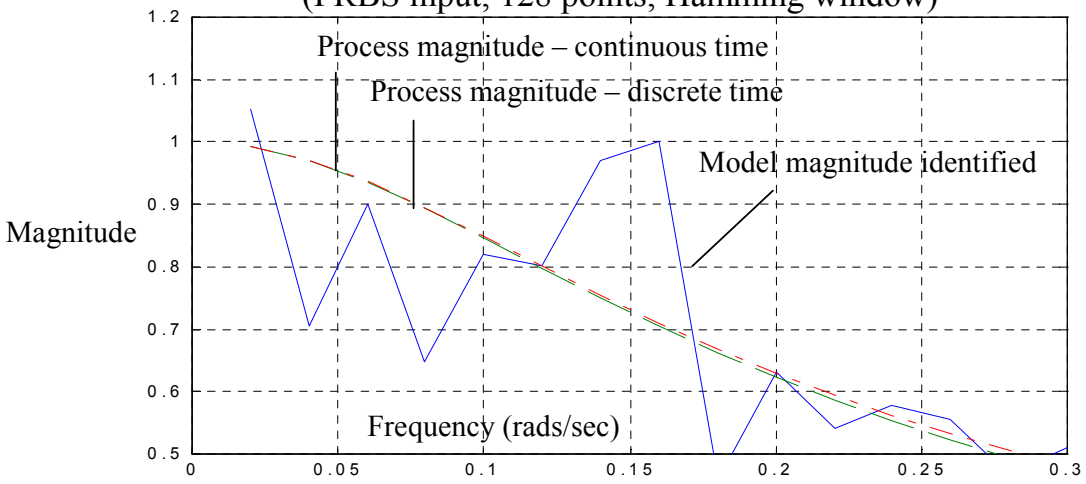
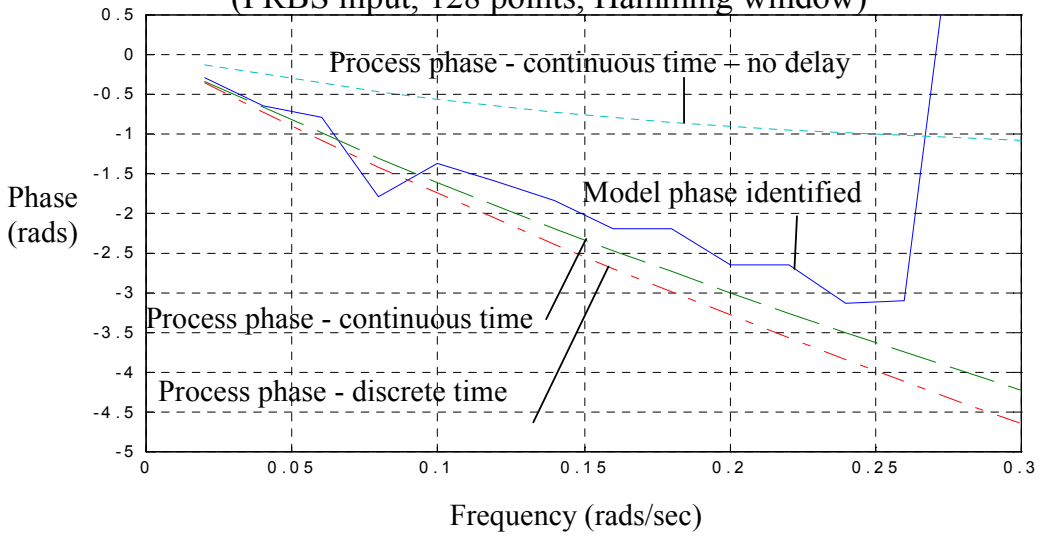



Figure 6: Simulated process and model phase
(PRBS input, 128 points, Hamming window)




***Good estimates*** of the process magnitude and phase are obtained. The estimates of the process magnitude and phase improve as the number of data points taken to find the bispectral magnitude and phase increase, as expected.

**Simulation results – cross-bispectral method – closed loop**

A simulation result that illustrates the estimation of the process magnitude and phase terms from the cross-bispectrum of the input and output signals, when the process is in closed loop regulator mode, under the control of a PID controller, is shown in Figures 7 and 8. The process is driven by an external exciting PRBS signal. The process transfer function taken is given by $e^{-0.833s}/(s+1)$; a Hamming window is used in the simulations. The PID controller used is tuned using the process reaction curve method of Ziegler and Nichols (1943).

Figure 7: Simulated process and model magnitude
(PRBS input, 64 points, Hamming window, Gaussian noise present)

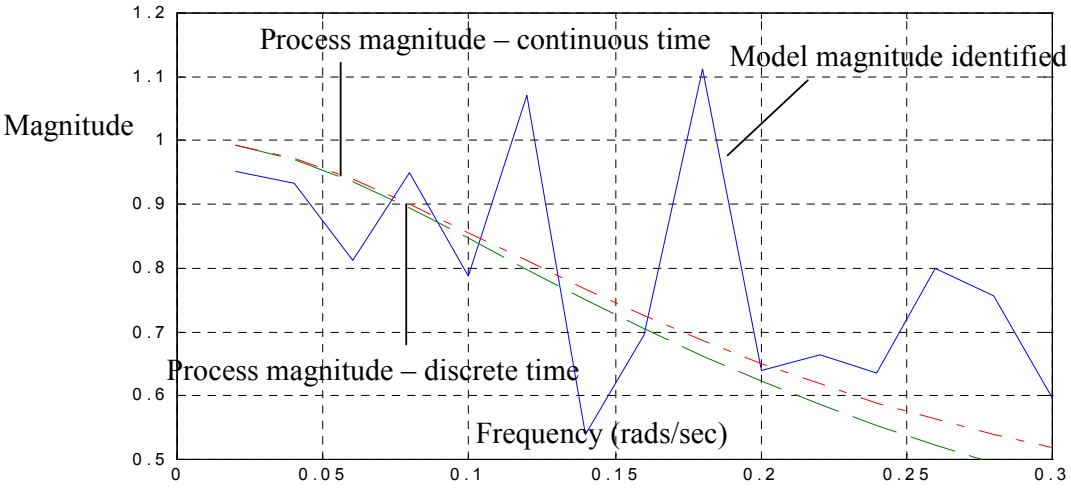
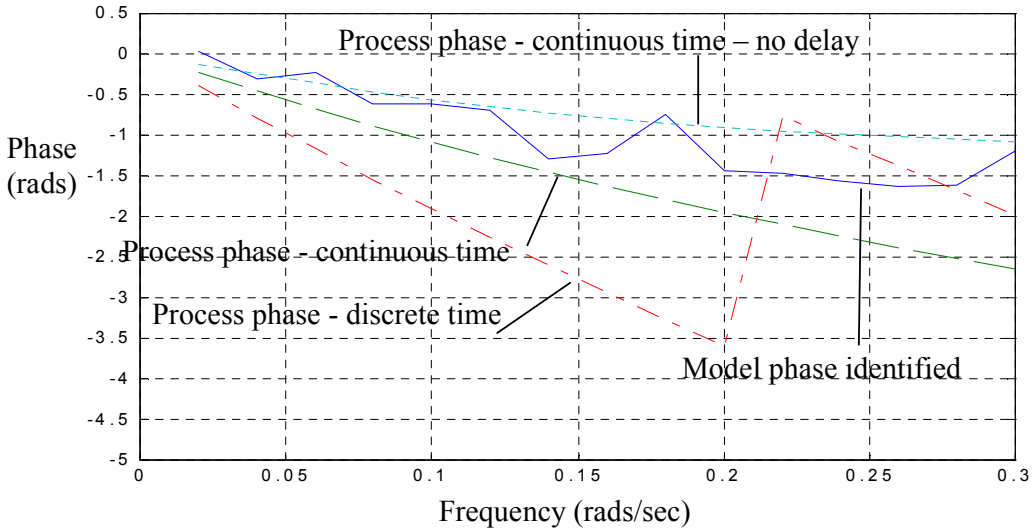



Figure 8: Simulated process and model phase
(PRBS input, 64 points, Hamming window, Gaussian noise present)




***Good estimates*** are obtained of the process magnitude and phase terms (both when Gaussian noise is present on the input and output signal, and when it is absent). The quality of the estimates of the process magnitude and phase terms are similar to those found in the open loop method.

# 4. Conclusions and Future Work

The table below gives some comparative information:

| | Number of data points needed | Computational intensity | Closed loop identification possible? | Robustness |
|---|---|---|---|---|
| F[o/p] / F[i/p] | Low - perhaps 100 [38] | Low | Gain and Phase estimation- Biased result under most conditions | Poor - noise and driving signals must be uncorrelated |
| Power Spectral Density | Medium- perhaps 1000 [20] | Medium | Gain estimation only- biased results under most conditions | Poor - noise and driving signals must be uncorrelated. Variance may be infinite [37] |
| Higher order spectra | High - perhaps 4000 samples averaged over 100 runs [4] | High | Gain and Phase estimation - unbiased results possible | Good, though variance may be high |

***The following conclusions about the use of bispectral techniques for process parameter estimation may be drawn:***

1. Conventional approaches for process frequency response estimation have a lower computational intensity and a requirement for a smaller number of data points than do the higher order spectral approaches. However, the higher order spectral approaches are robust to the presence of possibly mutually correlated, coloured Gaussian noise (or non-Gaussian noise, with a symmetric probability density function (p.d.f.)) added to both the process input and output.

2. The problem of process identification in closed loop using higher order spectra has not been completely resolved. The signals encountered in closed loop operation do not fit the requirement for the signals specified for process identification in all details; nevertheless, identification of the process

parameters may be possible in certain situations in a closed loop environment (e.g. if a PRBS driving signal is added to the input of the process), as the simulation results show.

The critical factor in the decision as to whether it is appropriate to use higher order spectra for process parameter estimation is the ***magnitude*** and ***nature*** of the ***additive noise present*** on both the input and output signals to the process.

***Possibilities for future work***

1. Identify both the process gain and phase (without the time delay contribution) using the bispectrum approach or the bicepstrum approach. Identify the time delay using an alternative method, and identify the non-time delay model parameters from the process gain or phase (without the time delay contribution) estimated.

2. Identify both the process gain and phase (with time delay contribution) using the cross-bispectrum approach. Calculate the time delay from the process phase characteristic and the non-time delay model parameters from the process gain characteristic.

3. Identify both the process gain and phase (without time delay contribution) using the bispectrum approach and the process gain and phase (with time delay contribution) using the cross-bispectrum approach. Identify the time delay from the difference in the two process phases (in this case, the time delay estimate will be approximately Gaussian distributed and will be approximately inversely proportional to the cube of the signal to noise ratio (SNR) [28]); subsequently identify the non-time delay model parameters.

4. Fit an ARMA model (in the z domain) to the process input and output data, and hence find the process magnitude and phase (using higher order spectral techniques). Then identify the model parameters (including time delay).

All of these methods rely on estimating the process gain and phase, from which the model parameters must be estimated. Alternatives to this approach are as follows:

1. Identify an overparameterised ARMA model in the z domain using a higher order spectral approach and then estimate the time delay and the other parameters from the ARMA model identified, by one of a number of well defined methods.

2. Estimate the parameters and the time delay of the process model by using a rational approximation for the time delay, identify the resulting ARMA model in the z domain (using a higher order spectral method) and subsequently estimate the time delay term.

3. Fit an ARMA model (in the z domain) to the process input and output data, and use model reduction techniques to find the parameters and time delay of a low order model plus time delay.

4. Identify the time delay term using a higher order spectral approach, and identify the non-time delay model parameters using an alternative approach; this may involve an iterative procedure.

5. Identify the non-time delay model parameters using a higher order spectral approach, and identify the time delay using an alternative approach; as above, this may involve an iterative procedure.

# References


1. Schwartzenbach, J. and Gill, K.F. (1984). *System Modelling and Control*, Second Edition, Edward Arnold.
2. O'Dwyer, A. (1998). "Frequency domain gradient methods of parameter and time delay estimation", *Technical Report AOD.98.03*, Dublin Institute of Technology, Dublin, Ireland.
3. Wellstead, P.E. (1986). *Lecture notes in control and information sciences, Vol. 79: Signal Processing for Control*. Editors: K. Godfrey, P. Jones. Series Editors: M. Thoma, A. Wyner. Springer-Verlag.
4. Nikias, C.L. and Petropulu, A.P. (1993). *Higher-order spectra analysis: a nonlinear signal processing framework*, Prentice-Hall Inc.
5. Mendel, J.M. (1991). Tutorial on higher-order statistics (spectra) in signal processing and system theory: theoretical results and some applications, *Proceedings of the IEEE*, **79**, 278-305.
6. Bartlet, H., Lohmann, A.W. and Wirnitzer, B. (1984). Phase and amplitude recovery from bispectra, *Applied Optics*, **23**, 3121-3129.
7. Matsuoka, T. and Ulrych, T.J. (1984). Phase estimation using the bispectrum, *Proceedings of the IEEE*, **72**, 1403-1411.
8. Haniff, C.A. (1991). Least-squares Fourier phase estimation from the modulo-$2\pi$ bispectrum phase, *Journal of the Optical Society of America A*, **8**, 134-140.
9. Matson, C.L. (1991). Weighted-least-squares magnitude-spectrum estimation from the bispectrum, *Journal of the Optical Society of America A*, **8**, 1914-1921.
10. Rangoussi, M. and Giannakis, G.B. (1991). FIR Modeling using log-bispectra: weighted least-squares algorithms and performance analysis, *IEEE Transactions on Circuits and Systems*, **38**, 281-296.
11. Cheng, F. and Venetsanopoulos, A.N. (1992). Determination of the exact phase of the bispectrum, *IEEE Transactions on Circuits and Systems - II: Analog and Digital Signal Processing*, **39**, 155-159.
12. Li, Y. and Ding, Z. (1994). A new nonparametric method for linear system phase recovery from bispectrum, *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, **41**, 415-419.
13. Alshebeili, S.A. and Cetin, A.E. (1990). A phase reconstruction algorithm from bispectrum, *IEEE Transactions on Geoscience and Remote Sensing*, **28**, 166-170.
14. Alshebeili, S.A., Cetin, A.E. and Venetsanopoulos, A.N. (1991). An adaptive system identification method based on bispectrum, *IEEE Transactions on Circuits and Systems*, **38**, 967-969.
15. Brooks, D.H. and Nikias, C.L. (1993). The cross-bicepstrum: definition, properties and application for simultaneous reconstruction of three nonminimum phase signals, *IEEE Transactions on Signal Processing*, **41**, 2389-2404.
16. Inouye, Y. and Tsuchiya, H. (1991). Identification of linear systems using input-output cumalents, *International Journal of Control*, **53**, 1431-1448.
17. Tugnait, J.K. (1992). Stochastic system identification with noisy input using cumalent statistics, *IEEE Transactions on Automatic Control*, **37**, 476-485.
18. Chen, J.-M. and Chen, B.-S. (1994). A higher order correlation method for model order and parameter estimation, *Automatica*, **30**, 1339-1344.
19. Inouye, Y. and Suga, Y. (1994). Identification of linear systems with noisy input using input-output cumalents, *International Journal of Control*, **59**, 1231-1253.
20. Delopoulos, A. and Giannakis, G.B. (1994). Consistent identification of stochastic linear systems with noisy input-output data, *Automatica*, **30**, 1271-1294.

21. Delopoulos, A. and Giannakis, G.B. (1996). Cumulant-based identification of noisy closed loop systems, *International Journal of Adaptive Control and Signal Processing*, **10**, 303-317.
22. Chen, J.-M. and Chen, B.-S. (1994). Identification of linear systems with unknown time delay and input-output noisy data: high-order correlation approach, *Control – Theory and Advanced Technology*, **10**, 3, 317-346.
23. Sasaki, K., Sato, T. and Makamura, Y. (1977). Holographic passive sonar, *IEEE Transactions on Sonics and Ultrasonics*, **SU-24**, 193-200.
24. Sato, T. and Sasaki, K. (1977). Bispectral holography, *Journal of the Acoustical Society of America*, **62**, 404-408.
25. Nikias, C.L. and Raghuveer, M.R. (1987). Bispectrum estimation: a digital signal processing framework, *Proceedings of the IEEE*, **75**, 869-891.
26. Nikias, C.L. and Pan, R. (1988). Time delay estimation in unknown Gaussian spatially correlated noise, *IEEE Transactions on Acoustics, Speech and Signal Processing*, **36**, 1706-1714.
27. Zhang, W. and Raghuveer, M. (1991). Nonparametric bispectrum-based time-delay estimators for multiple sensor data, *IEEE Transactions on Signal Processing*, **39**, 770-774.
28. Hinich, M.J. and Wilson, G.R. (1992). Time delay estimation using the cross bispectrum, *IEEE Transactions on Signal Processing*, **40**, 106-113.
29. Nikias, C.L. and Mendel, J.M. (1993). Signal processing with higher-order spectra, *IEEE Signal Processing Magazine*, **July**, 10-37.
30. Liang, Y.-C. and Leyman, A.R. (1997). Time delay estimation using higher order statistics, *Electronics Letters*, **33**, 9, 751-753.
31. Wu, Y. and Leyman, A.R. (1999). Time delay estimation in unknown spatially correlated Gaussion noises using higher-order statistics, Proceedings of the IEEE *International Conference on Acoustics, Speech and Signal Processing*, **5**, 2813-2816.
32. Tugnait, J.K. (1991). On time delay estimation with unknown spatially correlated Gaussian noise using fourth-order cumalents and cross cumalents, *IEEE Transactions on Signal Processing*, **39**, 1258-1267.
33. Reddy, G.R. and Rao, V.V. (1987). Signal delay and waveform estimation through differential cepstrum averaging, *IEEE Transactions on Acoustics, Speech and Signal Processing*, **ASSP-35**, 1487-1489.
34. Petropulu, A., Nikias, C.L. and Proakis, J.G. (1988). Cumalent cepstrum of FM signals and high-resolution time delay estimation, *Proceedings of the International Conference on Acoustics, Speech and Signal Processing*, 2642-2645, New York, USA.
35. Chiang, H.-H. and Nikias, C.L. (1990). A new method for adaptive time delay estimation for non-Gaussian signals, *IEEE Transactions on Acoustics, Speech and Signal Processing*, **38**, 209-219.
36. Tugnait, J.K. (1996). Frequency domain adaptive filters using higher order statistics with application to adaptive time delay estimation, *International Journal of Adaptive Control and Signal Processing*, **10**, 137-157.
37. Douce, P.E. and Balmor, L. (1990). Statistics of frequency-response estimates*, IEE Proceedings, Part D*, **137**, 290-296.
38. Ringwood, J.V. and O'Dwyer, A. (1994). A frequency domain based self tuning PID controller, *Proceedings of the Asian Control Conference*, 331-334, Tokyo, Japan.