



2005-01-01

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Recommended Citation

O'Connor, Niall and O'Dwyer, Aidan : Comparative study of two techniques for determining critical frequency response characteristics. Proceedings of the 4th Wismarer Automatisierungssymposium, Wismar, Germany, September 22-23, 2005.

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Comparative study of two techniques for determining critical system response characteristics

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Abstract: The paper presents a review of a comparative study of two separate techniques for obtaining important frequency and time domain characteristics of a system consisting of a process in series with a PI/PID controller in closed loop. The first technique involves the use of a Pseudo Random Binary Sequence (PRBS) to obtain the closed loop frequency response of the system. This closed loop frequency information may then be manipulated to obtain both open loop frequency response information and process model parameters. A further mathematical simulation using Matlab may then be performed independent of the actual process in order to obtain important time domain characteristics of the system such as rise time, settling time and overshoot. The second technique is a relay-based method described in [1], which may be used to obtain the open loop frequency response of the system. In this case, it is the open loop frequency response information that is manipulated in order to obtain closed loop frequency response information and process model parameters.

1.1 Introduction:

According to [2], in the testing of thousands of control loops in hundreds of operating plants, Techmation Inc. and others have found that more than 30% of the automatic control loops actually increase variability over manual control due to poor controller tuning. One reason why so many control loops perform poorly is that there are often numerous (more than a thousand) loops in a large process plant and not enough control engineers to maintain every loop. Jamsa-Jounela *et al.* [3] make the point that in order to ensure highest product quality it is essential to maintain the control system in an adequate manner. Vishnubhotla *et al.* [4] discuss how the current standard practice for industrial process control is to install DCS (Distributed Control Systems) and PLC control system platforms. These system platforms accumulate large volumes of process data, but there are very few data mining tools. As stated in [7], only about 20% of all control loops surveyed in mill audits have been found to actually reduce process variability in automatic mode. The remaining 80% of loops were found to increase variability. Of these, about 30% were found to oscillate due to control valve nonlinearities, another 30% performed poorly due to poor controller tuning and controller equipment design limitations, approximately 15% performed poorly due to deficiencies in control strategy design and about 5% performed poorly due to poor process design.

It is obvious, therefore, that there is a strong need for automatic assessment and monitoring of control loop performance. The goal of monitoring should be to provide information that can be used to assess the current status of the existing controller and to assist control engineers in deciding whether redesign is necessary [5]. When the controller performance is determined to be inadequate, it is important to ascertain whether an acceptable level of performance can be achieved with the existing control structure [6].

The paper presents two separate schemes for evaluating a control systems' performance. The first method presented is a PRBS based technique, which basically involves the application of a binary sequence test signal to the system under investigation and the subsequent employment of the Fast Fourier Transform (FFT) to the resulting system response. The second testing scheme, again involves the use of the FFT, however the system is excited through the use of a relay placed in series with the controller and process. For

each of the techniques discussed, a graphical user interface (GUI) was developed using Matlab 6.5 in order to provide a user-friendly environment in which the testing procedure could be carried out.

The paper is divided into the following sections. Section 1.2 provides a brief outline of the steps involved in each of the assessment schemes developed. The evaluation measures used to quantify control performance are defined in section 1.3. Results from a variety of simulations and experiments involving a process simulator are presented in section 1.4. Finally, a summary of the concepts discussed throughout the paper is provided in section 1.5.

1.2 Assessment Techniques:

The following section is designed to highlight the testing procedures used for both the PRBS based method and the relay based approach. The basic advantages and theory associated with each of the techniques will be presented in an effort to clarify their operational capacities. Also provided is theory behind the manipulation of frequency response information in order to obtain process model parameters.

1.2.1 PRBS Approach:

Pseudo random binary sequences (PRBS's), also known as pseudo noise (PN), linear feedback shift register (LFSR) sequences or maximal length binary sequences (m-sequences), are widely used in the field of system identification [8]. A pseudo random binary sequence, as its name suggests, is a semi-random binary sequence in the sense that it appears random within the sequence length, but the entire sequence repeats indefinitely. A PRBS sequence is an ideal test signal, as it simulates the random characteristics of a digital signal and can be easily generated. Pseudorandom binary sequences (PRBS's) are very effective as persistent excitation stimuli in dynamic testing [9]. Because the PRBS testing method is based on the cross-correlation techniques it is highly immune to extraneous noise of all kinds and as a result, its amplitude can be controlled to within safe limits without the risk of driving the plant outside the bounds of linear operation. The PRBS signal can also be easily coupled with the command input signal (set point) for normal plant operation. PRBS signal energy can be controlled over a range of frequencies with low amplitude by appropriate choice of the PRBS test signal parameters.

Correlation may be defined as a measure of similarity between two sequences. If the two sequences compared are different, 'crosscorrelation' is the term used and if they are the same, 'autocorrelation' is the term used. Mathematically, the autocorrelation of a sequence $x(k)$ of length L may be expressed as follows:

$$R_{xx}(m) = \frac{\sum_{k=1}^{L-m} (x(k) - \bar{x})(x(k+m) - \bar{x})}{\sum_{k=1}^L (x(k) - \bar{x})^2} \quad (1.2.1)$$

$$m = 1, 2, 3, \dots, L$$

For the case of a PRBS sequence, its 'cyclic' autocorrelation function has the following values:

$$R_{xx}(k\Delta t) = \begin{cases} V^2 & k = 0 \\ -\frac{V^2}{L} & k \neq 0 \end{cases} \quad (1.2.2)$$

where V is the bit interval voltage level, k is an integer and Δt is the pulse period (duration of each bit) of the PRBS. From (1.2.2) it can be seen that the autocorrelation function is a periodic triangular pulse train similar to that of the autocorrelation function of a truly random binary waveform. As the pulse period Δt vanishes and L becomes large the autocorrelation function tends closer to that of a periodic white noise source.

It has been well documented that the cross-correlation between the input $x(k)$ and the output $y(k)$ of a linear system, is related to the auto-correlation of the input by a convolution with the impulse response [10]:

$$\begin{aligned} y(k) &= h(k) \bullet x(k) \\ \Rightarrow x(k) * y(k) &= h(k) \bullet x(k) * x(k) \\ \Rightarrow R_{xy}(k) &= h(k) \bullet R_{xx}(k) \end{aligned} \quad (1.2.3)$$

where \bullet symbolises convolution and $*$ represents correlation. As already discussed, an important property of any PRBS is that its auto-correlation function is essentially an impulse. This impulse is represented by the Dirac delta function:

$$R_{xx}(k) \approx \delta(k) \quad (1.2.4)$$

The result of convolving a sequence with a Dirac delta function is the sequence itself. Thus the impulse response $h(k)$ can be found by cross-correlating the PRBS input $x(k)$ with the output $y(k)$:

$$R_{xy}(k) = \delta(k) \bullet h(k) = h(k) \quad (1.2.5)$$

Hence it is possible to measure the impulse response of linear systems by calculating the cross-correlation between the PRBS and the system output signal. The systems frequency

response may then be determined by applying the FFT to the systems impulse response.

1.2.2 Relay Based Approach:

There are a number of interesting features associated with relay based system testing. Two main advantages with this form of testing are 1) it is a closed loop test, which keeps the process variable under control and is usually preferred to open loop tests, and 2) a linear stable process with relay feedback is likely to automatically reach a sustained stationary oscillation known as a 'limit cycle'. Using the amplitude and period of this oscillation, information about the process critical point (point on the frequency response curve when input/output phase ratio is at -180 degrees) can easily be determined.

There are a number of different structures in which a relay feedback system (RFS) may be set up in order to obtain a control systems' critical response information. The most efficient of these relay-based methods appears to be the 'Weighting' method as discussed in [1]. This technique involves applying a decay weighting on the signals such that the weighted signals die off as time passes. This technique is also known as windowing. If $y(t)$ and $u(t)$ are the process output and relay output in a relay feedback system, a decaying exponential $e^{-\alpha t}$ ($\alpha > 0$) may be introduced to moderate these signals. Figure 1 illustrates how a typical relay output $u(t)$ is affected when the decaying exponential is applied to it, thus producing weighted output $\tilde{u}(t)$.

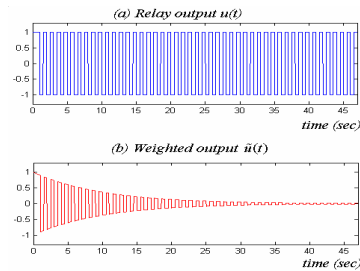


Figure 1 - Relay output and weighted version of relay output

The shifted process frequency response $G(j\omega_l + \alpha)$ may be determined using the following formula:

$$G(j\omega_l + \alpha) = \frac{Y(j\omega_l + \alpha)}{U(j\omega_l + \alpha)} = \frac{FFT\{\tilde{y}(kT)\}}{FFT\{\tilde{u}(kT)\}}, \quad (1.2.6)$$

$$l = 1, 2, 3, \dots, \frac{N}{2}$$

where $\omega_l = 2\pi l / (NT_s)$, $\tilde{y}(kT)$ and $\tilde{u}(kT)$ are the weighted system output and relay output respectively, N is the total number of samples taken of the output and T_s is the sampling period. According to [1], this weighting method yields the best results when compared with the other relay-based approaches.

1.2.3 Manipulating Frequency Response Information:

Consider the simple feedback system of Figure 2

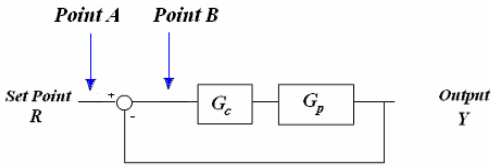


Figure 2 – Simple feedback system

For the case of the PRBS system-testing technique the binary testing sequence would be superimposed onto the setpoint and thus injected into the system at point A of Figure 2. The signals of interest in this testing procedure are therefore the PRBS signal and the system output, Y. After applying the FFT to these signals and determining the systems frequency response, as discussed in section 1.2.1, we are left with the closed loop frequency response, $M(s)$, of the control system.

$$M(s) = \frac{G_c(s)G_p(s)}{1 + G_c(s)G_p(s)} \quad (1.2.7)$$

While this is useful in itself, it is also possible to determine the open loop frequency response, $G_c(s)G_p(s)$, using this closed loop data. This may be done by applying the following formula:

$$G_c(s)G_p(s) = \frac{M(s)}{1 - M(s)} \quad (1.2.8)$$

Now considering the case of the relay-based approach, the relay is inserted at point B of Figure 2 and it is the open loop frequency response of the system that is obtained. In a similar fashion to that of the PRBS case, it is possible to obtain the closed loop frequency response of the system by simply applying equation (1.2.7). Using both the open loop and closed loop frequency response, important characteristics of the system may be determined. These frequency domain characteristics will be discussed further in section 1.3.

For both the PRBS based approach and the relay based technique, a least squares based method known as the Gradient approach [13] is used, in conjunction with the control systems open loop frequency response, to obtain the process model parameters for a First Order Lag Plus Dead-time (FOLPD) model. This FOLPD model takes the form

$$G_p(s) = \frac{K_m}{sT_c + 1} e^{-sT_d} \quad (1.2.9)$$

where the static gain K_m , model time constant T_c and model time delay T_d are determined through the use of the Gradient approach. A further mathematical simulation using Matlab may then performed independent of the actual process (using the process model determined through the gradient approach)

in order to obtain important time domain characteristics of the control system.

1.3 Evaluation Measures:

In order to quantify a control systems performance, a number of assessment measures were identified. These measures were considered appropriate as they provided a straightforward and easily interoperable measure of performance, and in most cases, these measures were industrial standard means of quantifying performance. These evaluation measures may be divided into two categories, namely time domain measures and frequency domain measures. While the time domain measures tend to provide a straightforward indication as to the response times of a system, the frequency domain measures tend to focus on system stability.

The time domain measures considered include the following: rise time, settling time, offset and overshoot. The rise time of a system can be defined as the time taken for the step response of a system to change from 10% to 90% of its final steady state value. A short rise time is usually desired. The settling time (T_s) is defined as the time the system takes to attain a ‘nearly constant’ value, usually +/- 5 percent of its final value [11]. Again, a short settling time is usually desired. Offset can be defined as the difference between the final, steady state value of the set point and that of the system output. In most cases, a zero steady state offset is desired [11]. The overshoot is the maximum amount that the system output exceeds its final steady state value and is usually expressed as a percentage.

The frequency domain measures decided upon include gain margin (Gm), phase margin (Pm), closed loop log modulus (L_{cmax}) and closed loop bandwidth (Bw). Both Pm and Gm are a direct measure of how much the phase and gain of the open loop system may vary before the closed loop system becomes unstable. While the phase and gain margin specifications can sometimes give poor results when the shape of the frequency response curve is unusual, the maximum closed loop log modulus does not have this problem. It is related to the closeness of the open loop frequency domain transfer function to the (-1,0) point at all frequencies [12]. The maximum closed loop log modulus is basically the peak value of the closed loop frequency response of the control system. A systems closed loop bandwidth is a direct measure of the response time capabilities of a system.

1.4 Results:

Table 1 provides an illustration of the type of results obtained using both the PRBS and relay based evaluation methods. The results of Table 1 were obtained when testing a process of three consecutive 1-second lags with a process gain of 1:

$$Process = \frac{1}{(s+1)(s+1)(s+1)} \quad (1.3.1)$$

This process was implemented using a Process Control Simulator PCS327 Mk2. For this example the controller was considered to be a proportional controller with a gain of 1. The percentage error was calculated as 100% times the relative error:

$$\partial x = \frac{x_0 - x}{x} * 100\% = \left(\frac{x_0}{x} - 1 \right) * 100\% \quad (1.3.2)$$

where x was the sum of the absolute ‘actual’ results and x_0 was the sum of the absolute measured results. As can be seen from the table the PRBS based evaluation method proved to be consistently accurate when estimating frequency domain measures whereas the relay based approach appeared to be profoundly affected by noise levels. It should also be noted that the majority of the error associated with the PRBS based method came as a result of modeling the actual 3rd

order process as a FOLPD model and then estimating the time domain characteristics based on this model. If a higher order model was used this error could be greatly reduced.

1.5 Summary:

This paper presented a review of the concepts associated with two independent methods for evaluating important time and frequency domain characteristics of a control system. Following a section outlining the theory and properties associated with each technique, the results of a comparative process test were provided. From these results it was obvious that the PRBS based testing scheme proved more reliable and more accurate, in the majority of instances, especially with increased noise levels.

Table 1 – Results obtained from a Matlab based Experiment

	Ts (secs)	Tr (secs)	Overshoot (%)	Offset (%)	Time error (%)	Gm (dBs)	Pm	Lcmax (dBs)	Bw (rads)	Frequency Error (%)	Noise (%)	Km	Tc (secs)	Td (secs)
Actual	5.79	1.87	13.9	50	-	18.1	0	-5.24	1.17	-	-	-	-	-
PRBS	3.1	1.76	0	53.64	-18.25%	18.68	0	-5.83	1	4.08%	0	0.88	2.49	0.84
Relay	3.78	1.2	5.79	52.37	-11.77%	17.61	0	-5.75	1.2	0.20%		0.91	1.99	0.96
PRBS	3.07	1.74	0	54.43	-17.22%	18.29	0	-5.87	1	2.65%	5	0.85	2.4	0.81
Relay	2.59	1.43	0.87	40.9	-36.01%	20.97	122	-4.77	1.07	507.14%		1.41	2.86	0.79
PRBS	2.9	1.64	0	55.4	-16.24%	18.15	0	-5.86	1	2.04%	10	0.85	2.38	0.79
Relay	2.99	1.72	0	38.37	-39.80%	20.33	93.83	-3.14	0.13	379.11%		1.62	3.6	0.77
PRBS	2.9	1.64	0	55.26	-16.43%	18.07	0	-5.77	1	1.35%	15	0.85	2.34	0.79
Relay	2.6	1.45	1.07	30.26	-50.56%	21.43	87.59	-2.21	0.78	357.00%		2.22	4.09	0.78
PRBS	2.9	1.66	0	55.25	-16.42%	17.92	0	-5.89	1	1.22%	20	0.85	2.37	0.8
Relay	3.41	2.18	0	62.9	-4.29%	1.06	0	-5.35	1.4	-68.14%		0.59	1.92	0.46
PRBS	2.88	1.65	0	55.36	-16.31%	17.86	0	-5.89	1	0.98%	25	0.85	2.35	0.79
Relay	3.53	2.19	0	66.66	1.15%	26.59	0	-5.7	1.4	37.45%		0.5	1.87	0.58
PRBS	2.87	1.63	0	55.28	-16.46%	17.95	0	-5.74	1	0.73%	30	0.84	2.31	0.79
Relay	2.28	1.15	0.13	72.14	5.79%	17.04	0	-5.99	1.54	0.24%		0.38	1.09	0.8
PRBS	2.9	1.63	0	55.58	-16.00%	17.87	0	-5.87	1	0.94%	35	0.84	2.34	0.79
Relay	2.08	0.46	12.7	71.8	21.63%	27	0	-5.4	4.7	51.37%		0.39	0.48	0.63
PRBS	2.9	1.64	0	55.33	-16.34%	17.74	0	-5.9	1	0.53%	40	0.85	2.35	0.79
Relay	0.58	0.42	0	684.12	857.41%	4.5	44.8	6.55	33.79	265.73%		3.07	0.07	0.13

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