Vertical Response of a Footbridge Subjected to Stochastic Crowd Loading

Joseph Keogh

Dublin Institute of Technology, School of Civil & Building Services Engineering, Bolton Street, Dublin, Ireland

1. Introduction

- Modern developments in the design of structures and progress in structural materials have led to longer and lighter footbridges
- These typically low frequency bridges can experience vibration problems resulting from the dynamic nature of pedestrian load application
- Synchronization is a well known phenomenon in crowd loading and is evident in everyday use of a bridge but in particular with troops marching
- A catastrophic case of this occurred in Anger, France (1850) where 200 troops died when the bridge they were marching across (The Basse-Chaîne Bridge) collapsed due to excessive vibrations
- This prompted the erection of signs on bridges asking troops to break step while cross a bridge, Fig 1 is from the Albert Bridge in London
- A well known recent example of this phenomenon is the Millennium Bridge, London (Fig 2) which closed days after opening due to excessive vibrations
- In this work, a model is created to allow prediction of the vertical response of a footbridge subjected to single pedestrian and crowd loading.

Fig 1. Notice to Reduce Synchronization Problems

Fig 2. Millennium Bridge, London

2. Modelling Procedure

Step 1: Modelling Human Foot Force

When walking a pedestrian produces forces in three directions (vertical, lateral and longitudinal), to date, only the vertical force (Fig 2(a)) is considered in this work. The force of the left and right foot overlap in time during walking (Fig 2(b)), the sum of which can be represented by a Fourier series (shown below)

$$P(t) = m_P g[1 + r\sin(2\pi f_p t)]$$

where; *P(t)* is the time varying force (N), m_p is the pedestrian weight (kg), g is gravity (m/s²), *r* is a Fourier coefficient, f_p is the pacing frequency (Hz) and *t* is the time (s).

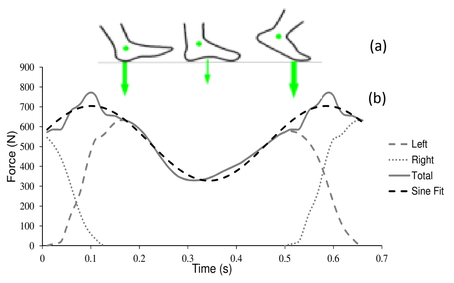

Fig 2. Pedestrian Foot Force

Step 2: Statistical Pedestrian Parameters

Unlike current design codes, the model generated by this research incorporates statistical distributions of pedestrian parameters (Fig 3). The model allows prediction of vibration response resulting from non-homogenous single pedestrians and crowds of pedestrians with varying levels of synchronization (0 to 100%). The level of synchronization, that is people walking with the same pacing frequency and phase angle, may be increased by people marching, walking in groups or the vibration of the bridge forcing pedestrians to walk in a certain manner, as was the case on the London Millennium Bridge.

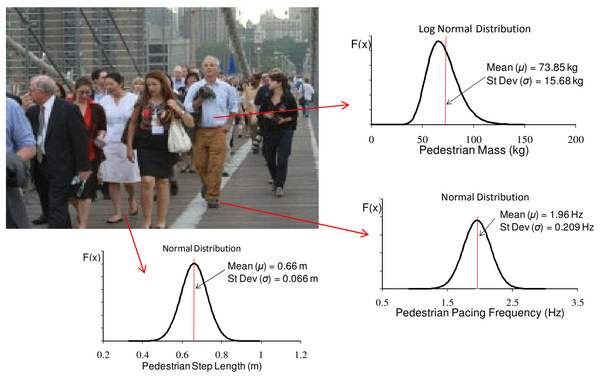

Fig 3. Non-homogeneous Crowd of Pedestrians

Step 3: Finite Element Modelling

To establish a vibration response a finite element model of a hypothetical bridge was developed in Matlab. The beam was modelled using 10 Euler-Bernoulli beam elements with lumped mass assumed. Transient solutions were obtained using Newmark-β method. The force defined in Step 1 was moved across the bridge at a velocity equal to the product of the pacing frequency and step length.

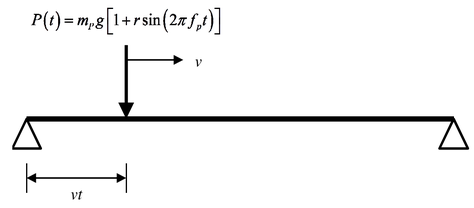

Fig 4. Moving Force Model

3. Model Results

- The response of interest in this study is the mid-span acceleration
- Response is assessed using a 5 s root-mean-square
- Characteristic response is the response with a 5% probability of exceedance following 1000 simulations
- Fig 5(a) shows the vibration response for a typical crowd
- Fig 5(b) shows the number of pedestrians on the bridge against time, both total and those synchronized
- Fig 5(c) shows the time at which each pedestrian enters and leaves the bridge

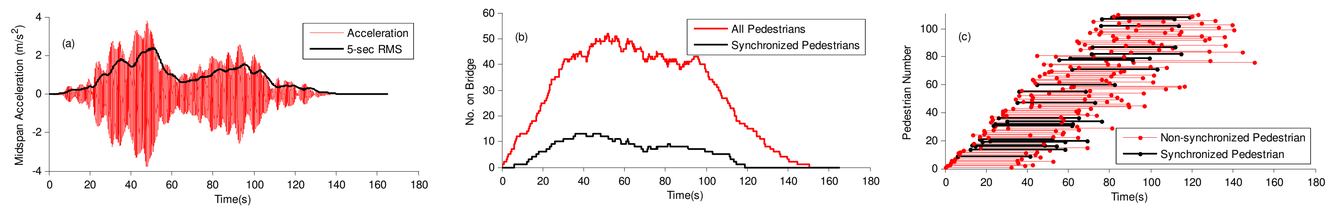

Fig 5. Crowd Response and Diagnostics – Density = 0.5 p/m², Synchronization = 20%

4. Accuracy of Current Guidelines

Fig 6 shows the model results of a low frequency footbridge (2.17 Hz) subjected to loading from a typical crowd (density of 0.5 persons/metre²) with varying levels of synchronization. This is compared to current design codes and guidelines, the predictions of which are seen to vary by as much as a factor of four.

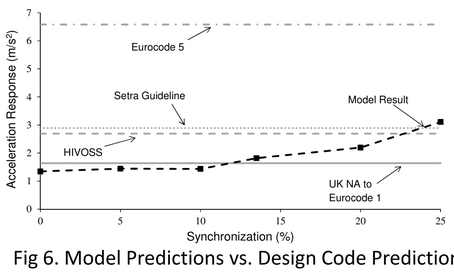

Fig 6. Model Predictions vs. Design Code Predictions

5. Spring Mass Damper Model

The moving force model does not allow for any possible interaction between the pedestrian and bridge, denoted Human Structure Interaction (HSI). This phenomenon is found to be of particular importance if the pedestrian pacing frequency matches, or is close to, the bridge natural frequency. To allow for this interaction, a Spring Mass Damper (Fig 7) is incorporated into the model. The ratio of moving spring mass damper model response to the moving force model response for a single pedestrian is presented in Fig 8 for three different bridge frequencies. A significant reduction is evident when the bridge frequency is close to the mean pacing frequency of 1.96 Hz.

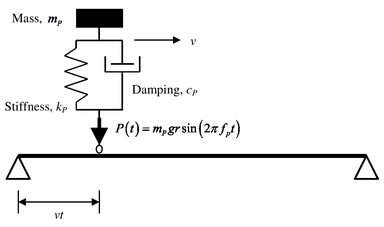

Fig 7. Spring Mass Damper

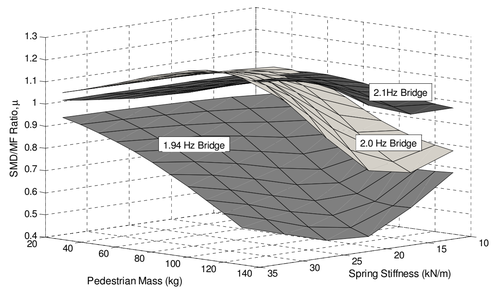

Fig 8. Ratio of Spring Mass Damper to Moving Force for Three Bridges

6. Conclusions

- A statistical model is developed for the prediction of vibration response resulting from the passage of non-homogeneous single pedestrians and crowds of pedestrians of varying synchronization levels
- The accuracy of the model is proven with comparison to design codes and other literature reports
- Vibration response is proven to be very sensitive to pacing frequency and is found to be a function of both crowd density and synchronization level
- A spring mass damper is incorporated into the model to account for human structure interaction
- The effectiveness of the spring mass damper is most prominent when the bridge frequency is close to the pacing frequency

7. Future Work

- Laboratory static tests will investigate human structure interaction by loading the bridge with a static pedestrian and an equivalent dead mass at mid-span (Fig 9(a))
- Further tests will investigate vibration response of walking pedestrians and pedestrians carrying addition mass (Fig 9(b))
- The aim of this is to assess the best model; moving force, moving mass or moving spring mass damper, to represent a pedestrian crossing a flexible footbridge (Fig 9(c))

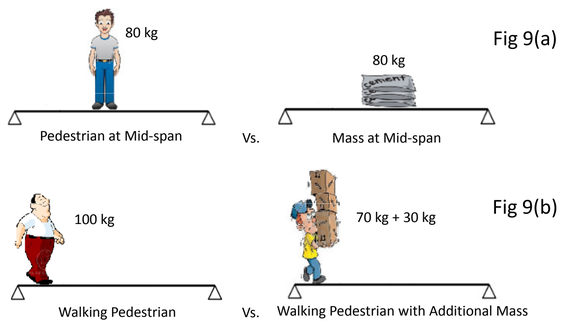

Fig 9(a)

Fig 9(b)

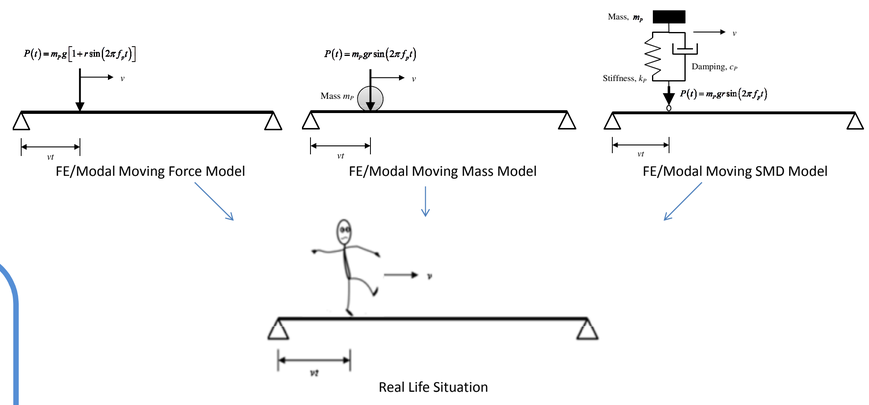

Fig 9(c)

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