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PI and PID Controller Tuning Rule Design for Processes with Delay, to achieve Constant Gain and Phase Margins for all Values of Delay

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Abstract

This paper will discuss the design of PI and PID controller tuning rules to compensate processes with delay, that are modelled in a number of ways. The rules allow the achieveme nt of constant gain and phase margins as the delay varies.

1. INTRODUCTION

The ability of PI and PID controllers to compensate most practical industrial processes has led to their wide acceptance in industrial applications. The requirement to choose eit her two or three controller parameters has meant that the use of tuning rules to determine these parameters are popular. The author has previously considered this topic in detail [1-4]. Normally, the gain and phase margins of the compensated systems tend to increase as the time delay increases, reflecting the common view that PI and PID controllers are less suitable for the control of dominant time delay processes; however, the author discovered that a number of PI tuning rules, in particular, had the chara cteristic of allowing constant gain and phase margin, as the delay varied, for processes modelled in first order lag plus delay (FOLPD) form [5]. This paper proposes an original approach to design tuning rules for both PI and PID controllers, for a wider variety of process models, with the above characteristic.

The paper is organised as follows. In Section 2, PI controller tuning rules are specified for processes modelled in FOLPD form and integral plus delay (IPD) form. In Section 3, PID controller tuning rules are described for processes modelled in FOLPD form, second order system plus delay (SOSPD) form and SOSPD form with a negative zero. Section 4 deals with the design of PD controller tuning rules for the control of processes modelled in first order la g plus integral plus delay (FOLIPD) form. Finally, Section 5 contains some concluding remarks.

2. PI CONTROLLER DESIGN

2.1 Processes modelled in first order lag plus delay (FOLPD) form

For such processes and controllers, $G_m(s) = \dfrac{K_m e^{-s\tau_m}}{1 + sT_m}$ (1)

with $$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right) \quad (2)$$

Then, $G_m(j\omega)G_c(j\omega) = \dfrac{K_m e^{-j\omega\tau_m}}{1 + j\omega T_m}\dfrac{K_c(j\omega T_i + 1)}{j\omega T_i}$, with

ω_g being found from $\left|G_m(j\omega_g)G_c(j\omega_g)\right| = 1$ and

ω_p found from $\angle G_m(j\omega_p)G_c(j\omega_p) = -\pi$.

Then, $G_m(j\omega)G_c(j\omega) = \dfrac{K_m K_c\sqrt{1+\omega^2 T_i^2}}{\omega T_i\sqrt{1+\omega^2 T_m^2}}\angle -0.5\pi + \tan^{-1}\omega T_i - \tan^{-1}\omega T_m - \omega\tau_m$.

The phase margin, $$\phi_m = \pi - 0.5\pi + \tan^{-1}\omega_g T_i - \tan^{-1}\omega_g T_m - \omega_g\tau_m \quad (3)$$

with ω_g given by the solution of $$\omega_g = \frac{K_m K_c\sqrt{1+\omega_g^2 T_i^2}}{\omega_g T_i\sqrt{1+\omega_g^2 T_m^2}} = 1 \quad (4)$$

The gain margin, $$A_m = \frac{1}{\left|G_m(j\omega_p)G_c(j\omega_p)\right|} = \frac{\omega_p T_i}{K_m K_c}\sqrt{\frac{1+\omega_p^2 T_m^2}{1+\omega_p^2 T_i^2}} \quad (5)$$

with ω_p given by the solution of $$\omega_p = -0.5\pi + \tan^{-1}\omega_p T_i - \tan^{-1}\omega_p T_m - \omega_p \tau_m = -\pi \quad (6)$$

If K_c and T_i are designed as follows: $$K_c = \frac{aT_m}{K_m \tau_m} \quad (7)$$

and $$T_i = T_m \quad (8)$$

Then equation (6) becomes $-0.5\pi - \omega_p \tau_m = -\pi$ i.e. $\omega_p = \pi/2\tau_m$. Substituting into equation (5) gives $A_m = \pi T_m/2K_m K_c \tau_m$. Equation (4) becomes $K_m K_c/\omega_g T_m = 1$ i.e. $\omega_g = K_m K_c/T_m$. Equation (3) then becomes $\phi_m = 0.5\pi - K_m K_c \tau_m/T_m$. Then, from equation (7),

$$A_m = \pi/2a \quad (9)$$

and $$\phi_m = 0.5\pi - a \quad (10)$$

Some typical tuning rules are shown in the table below.

Table 1: Typical PI controller tuning rules – FOLPD process model

a	K_c	T_i	A_m	ϕ_m
$\pi/3$	$1.047\,T_m/K_m\tau_m$	T_m	1.5	$\pi/6$
$\pi/4$	$0.785T_m/K_m\tau_m$	T_m	2.0	$\pi/4$
$\pi/6$	$0.524T_m/K_m\tau_m$	T_m	3.0	$\pi/3$

It may also be demonstrated that, if K_c and T_i are designed as follows: $$K_c = \frac{aT_m}{\tau_m}\frac{T_u K_u}{\sqrt{T_u^2 + 4\pi^2 T_m^2}} \quad (11)$$

and $$T_i = T_m \quad (12)$$

where K_u and T_u are the ultimate gain and ultimate period, respectively, then the constant gain and phase margins provided in equations (9) and (10) are obtained.

2.2 Processes modelled in integral plus delay (IPD) form

A similar analysis to that of Section 2.1 may be done for the design of PI controllers for processes modelled in integral plus delay (IPD) form. The process is modelled as follows: $$G_m(s) = K_m e^{-s\tau_m}/s \quad (13)$$

Corresponding to equations (3) and (4), the phase margin is

$$\phi_m = \pi - 0.5\pi + \tan^{-1}\omega_g T_i - 0.5\pi - \omega_g \tau_m \quad (14)$$

with ω_g given by the solution of $$\omega_g = \frac{K_m K_c\sqrt{1+\omega_g^2 T_i^2}}{\omega_g^2 T_i} = 1 \quad (15)$$

If K_c and T_i are designed as follows: $$K_c = \frac{a}{K_m \tau_m} \quad (16)$$

and $$T_i = b\tau_m \quad (17)$$

then it may be shown that $$\omega_g = \left[\frac{a^2}{2\tau_m^2} \pm \frac{a}{2b\tau_m^2}\sqrt{a^2 b^2 + 4}\right]^{0.5} \quad (18)$$

and, from (14), $$\phi_m = \tan^{-1}\left[0.5a^2b^2 + 0.5ab\sqrt{a^2b^2+4}\right]^{0.5} - \sqrt{0.5a^2 + 0.5\frac{a}{b}\sqrt{a^2b^2+4}} \quad (19)$$

Corresponding to equations (5) and (6), the gain margin,

$$A_m = \frac{\omega_p^2 T_i}{K_m K_c\sqrt{1+\omega_p^2 T_i^2}} \quad (20)$$

with ω_p given by the solution of $$\omega_p = -0.5\pi + \tan^{-1}\omega_p T_i - 0.5\pi - \omega_p \tau_m = -\pi \quad (21)$$

i.e. ω_p is given by $\tan^{-1}\omega_p T_i = \omega_p \tau_m$. An analytical solution to this equation is not possible, though if the approximation $\tan^{-1}\omega_p T_i \approx 0.5\pi - \frac{\pi}{4\omega_p T_i}$ (when $|\omega_p T_i| > 1$) is used, simple calculations show that the following analytical solution for ω_p may be obtained:

$$\omega_p = \frac{0.5}{\tau_m}\left[0.5\pi + \sqrt{0.25\pi^2 - \frac{\pi}{b}}\right] \tag{22}$$

The inequality $|\omega_p T_i| > 1$ may be shown to be equivalent to b > 1.273. From (20) and (22), calculations show:

$$A_m = \frac{\frac{b}{4a}\left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{b}}\right]^2}{\sqrt{1 + \frac{b^2}{4}\left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{b}}\right]^2}} \tag{23}$$

Some typical tuning rules are shown in the table below.

Table 2: Typical PI controller tuning rules – IPD process model

K_c	T_i	A_m	ϕ_m
$0.558/K_m\tau_m$	$1.4\tau_m$	1.5	46.2^0
$0.484/K_m\tau_m$	$1.55\tau_m$	2.0	45.5^0
$0.458/K_m\tau_m$	$3.35\tau_m$	3.0	59.9^0
$0.357/K_m\tau_m$	$4.3\tau_m$	4.0	60.0^0
$0.305/K_m\tau_m$	$12.15\tau_m$	5.0	75.0^0

It may also be shown that the maximum sensitivity (which is the shortest distance from the Nyquist curve to the (-1,0) point on the Rl-Im axis) is a constant using the above tuning strategy. The maximum sensitivity is calculated as:

$$\text{Maximum sensitivity} = \left[1 - 2a\cos(\omega_r\tau_m) - \frac{a^2}{\omega_r^2\tau_m^2}\right]^{-0.5} \tag{24}$$

with $\omega_r\tau_m$ being a constant obtained numerically from $(a^2/\omega_r^2\tau_m^2) + \cos\omega_r\tau_m = \sin(\omega_r\tau_m)/\omega_r\tau_m$.

It may also be shown that, if K_c and T_i are designed as follows:

$$K_c = aK_u \tag{25}$$

and

$$T_i = bT_u \tag{26}$$

then the following constant gain and phase margins are determined:

$$A_m = \frac{\frac{2b}{\pi a}\left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{4b}}\right]^2}{\sqrt{1 + \frac{1}{a^2\pi^2}\left[\frac{\pi}{2} + \sqrt{\frac{\pi^2}{4} - \frac{\pi}{4b}}\right]^2}} \tag{27}$$

and

$$\phi_m = \tan^{-1}\left[2\pi^2a^2b^2 + 2\pi ab\sqrt{4\pi^2a^2b^2 + 4}\right]^{0.5} - \sqrt{0.125\pi^2a^2 + \frac{\pi a}{16b}\sqrt{4\pi^2a^2b^2 + 4}} \tag{28}$$

3. PID CONTROLLER DESIGN

3.1 Process modelled in FOLPD form; Classical PID controller

The classical PID controller is given by

$$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)\left(\frac{1 + sT_d}{1 + s\alpha T_d}\right) \tag{29}$$

and the process model is given by equation (1). Following the procedure in Section 2.1, it may be shown that

$$\phi_m = 0.5\pi + \tan^{-1}\omega_g T_i + \tan^{-1}\omega_g T_d - \tan^{-1}\omega_g T_m - \tan^{-1}\omega_g\alpha T_d - \omega_g\tau_m \tag{30}$$

and $$A_m = \frac{\omega_p T_i}{K_c K_m}\sqrt{\frac{(1+\omega_p{}^2 T_m{}^2)(1+\omega_p{}^2\alpha^2 T_d{}^2)}{(1+\omega_p{}^2 T_i^2)(1+\omega_p{}^2 T_d{}^2)}} \quad (31)$$

with ω_p given by $$0.5\pi + \tan^{-1}\omega_p T_d + \tan^{-1}\omega_p T_i - \tan^{-1}\omega_p T_m - \tan^{-1}\omega_p \alpha T_d - \omega_p \tau_m = 0 \quad (32)$$

and ω_g given by $$\frac{K_m K_c \sqrt{1+\omega_g{}^2 T_i{}^2}\sqrt{1+\omega_g{}^2 T_d{}^2}}{\omega_g T_i \sqrt{1+\omega_g{}^2 T_m{}^2}\sqrt{1+\alpha^2\omega_g{}^2 T_d{}^2}} = 1$$

(33)

If K_c, T_i and T_d are designed as follows: $$K_c = \frac{aT_m}{K_m \tau_m} \quad (34)$$

$$T_i = \alpha T_m \quad (35)$$

and $$T_d = T_m \quad (36)$$

then equation (32) becomes $0.5\pi - \omega_p \tau_m = 0$ i.e. $\omega_p = \pi/2\tau_m$, and equation (31) becomes $A_m = \alpha \pi T_m / 2K_m K_c \tau_m$. Equation (33) becomes $K_m K_c / \omega_g T_m = 1$ i.e. $\omega_g = K_m K_c / T_m$. Equation (30) then becomes $\phi_m = 0.5\pi - K_m K_c \tau_m / \alpha T_m$.

Then, from equation (34), $$A_m = \pi\alpha/2a \quad (37)$$

and $$\phi_m = 0.5\pi - a/\alpha \quad (38)$$

This design reduces to the PI controller design when $\alpha = 1$.

3.2 Process modelled in second order system plus delay (SOSPD) form; Series PID controller

For such processes and controllers, $$G_m(s) = \frac{K_m e^{-s\tau_m}}{(1+sT_{m1})(1+sT_{m2})} \quad (39)$$

with $$G_c(s) = K_c\left(1 + \frac{1}{T_i s}\right)(1+sT_d) \quad (40)$$

Therefore, $G_m(s)G_c(s) = \dfrac{K_m K_c e^{-s\tau_m}(1+sT_i)(1+sT_d)}{T_i s(1+sT_{m1})(1+sT_{m2})}$. If T_i and T_d are designed as follows:

$$T_i = T_{m1}$$

(41)

and $$T_d = T_{m2} \quad (42)$$

then $G_m(s)G_c(s) = \dfrac{K_m K_c e^{-s\tau_m}}{T_i s}$, which is equal to $G_m(s)G_c(s)$ in Section 2.1 when $T_i = T_m$. Therefore, designing

$$K_c = \frac{aT_m}{K_m \tau_m} \quad (43)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as before.

3.3 Process modelled in SOSPD form with a negative zero; Classical PID controller

For such processes, $$G_m(s) = \frac{K_m e^{-s\tau_m}(1+sT_{m3})}{(1+sT_{m1})(1+sT_{m2})} \quad (44)$$

with $G_c(s)$ given by equation (29). Therefore, $G_m(s)G_c(s) = \dfrac{K_m K_c e^{-s\tau_m}(1+sT_{m3})(1+sT_i)(1+sT_d)}{T_i s(1+sT_{m1})(1+sT_{m2})(1+\alpha s T_d)}$. T_i, T_d and α are designed as follows: $$T_i = T_{m1}$$

(45)

$$T_d = T_{m2} \quad (46)$$

and $$\alpha = T_{m3}/T_{m2} \quad (47)$$

Therefore, designing $$K_c = \frac{aT_m}{K_m \tau_m} \quad (48)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as in Sections 2.1 and 3.2.

4. PD CONTROLLER DESIGN

In this case, the process is modelled in first order lag plus integral plus delay (FOLIPD) form i.e.

$$G_m(s) = \frac{K_m e^{-s\tau_m}}{s(1+sT_m)} \quad (49)$$

with

$$G_c(s) = K_c(1+T_d s) \quad (50)$$

Therefore, $G_m(s)G_c(s) = \frac{K_m K_c e^{-s\tau_m}(1+sT_d)}{s(1+sT_m)}$. Therefore, designing

$$K_c = \frac{aT_m}{K_m \tau_m} \quad (51)$$

and

$$T_d = T_m \quad (52)$$

will allow $A_m = \pi/2a$ and $\phi_m = 0.5\pi - a$, as in Sections 2.1, 3.2 and 3.3.

5. DISCUSSION AND CONCLUSIONS

A number of authors have proposed tuning rules which have the effect of allowing the achievement of a constant gain and phase margin, as the time delay of the process model varies. For the PI control of a FOLPD process model, O'Dwyer [5] shows that the tuning rules proposed by Chien *et al*. [6], Haalman [7], Pemberton [8], Smith and Corripio [9], Rivera *et al*. [10], Fruehauf *et al*. [11], Hang *et al*. [12], Ho *et al*. [13], Voda and Landau [14], Cox *et al*. [15], St. Clair [16] and Bi *et al*. [17] facilitate the achievement of a constant gain and phase margin. Most of these tuning rules are determined from a time domain controller design approach. Haalman [7] proposes a tuning rule that gives a constant gain and phase margin when the process is modelled in FOLPD form, with the PID controller being in series form.

The paper discusses an original approach to design tuning rules for both PI and PID controllers, for a wide variety of process models that include a delay, with the primary objective of achieving constant gain and phase margins for all values of delay. In one of the cases discussed (PI control of an IPD process model), an analytical approximation is used in the development; this approximation may also be used to determine further tuning rules for other process models with delay, and other PID controller structures.

REFERENCES

[1] O'Dwyer, A. (1998). "Performance and robustness issues in the compensation of FOLPD processes with PI and PID controllers", Proceedings of the *Irish Signals and Systems Conference*, pp. 227-234, Dublin Institute of Technology, Kevin St., Dublin, June.

[2] O'Dwyer, A. (1999). "PI and PID controller tuning rules for time delay processes: a summary. Part 1: PI controller tuning rules", Proceedings of the *Irish Signals and Systems Conference*, pp. 331-338, National University of Ireland, Galway, June.

[3] O'Dwyer, A. (1999). "PI and PID controller tuning rules for time delay processes: a summary. Part 2: PID controller tuning rules", Proceedings of the *Irish Signals and Systems Conference*, pp. 339-346, National University of Ireland, Galway, June.

[4] O'Dwyer, A. (2000). "PID compensation of time delayed processes: a survey", Proceedings of the *Irish Signals and Systems Conference*, pp. 5-12, National University of Ireland, Dublin, June.

[5] O'Dwyer, A. (2000). "Performance and robustness of PI and PID tuning rules", *Technical Report AOD-00-12*, Dublin Institute of Technology, Ireland.

[6] Chien, K.L., Hrones, J.A. and Reswick, J.B. (1952). On the automatic control of generalised passive systems, *Transactions of the ASME*, **February**, 175-185.

[7] Haalman, A. (1965). Adjusting controllers for a deadtime process, *Control Engineering*, **July**, 71-73.

[8] Pemberton, T.J. (1972). PID: The logical control algorithm, *Control Engineering*, **19, 5**, 66-67.

[9] Smith, C.A. and Corripio, A.B. (1985). *Principles and practice of automatic process control*, John Wiley and Sons.

[10] Rivera, D.E., Morari, M. and Skogestad, S. (1986). Internal Model Control. 4. PID controller design, *Industrial and Engineering Chemistry Process Design and Development*, **25**, 252-265.

[11] Fruehauf, P.S., Chien, I.-L. and Lauritsen, M.D. (1993). Simplified IMC-PID tuning rules, *ISA/93 Advances in Instrumentation and Control*, **48**, 1745-1766, McCormick Place, Chicago, Illinois, USA.

[12] Hang, C.C., Lee, T.H. and Ho, W.K. (1993). *Adaptive Control*, Instrument Society of America.

[13] Ho, W.K., Hang, C.C. and Cao, L.S. (1995b). Tuning of PID controllers based on gain and phase margin specifications, *Automatica*, **31**, 497-502.

[14] Voda, A. and Landau, I.D. (1995). The autocalibration of PI controllers based on two frequency measurements, *International Journal of Adaptive Control and Signal Processing*, **9**, 395-421.

[15] Cox, C.S., Daniel, P.R. and Lowdon, A. (1997). Quicktune: a reliable automatic strategy for determining PI and PPI controller parameters using a FOLPD model, *Control Engineering Practice*, **5**, 10, 1463-1472.

[16] St. Clair, D.W. (1997). *Controller tuning and control loop performance*, 2nd Edition, Straight Line Control Co., Inc.

[17] Bi, Q., Cai, W.-J., Lee, E.-L., Wang, Q.-G., Hang, C.-C. and Zhang, Y. (1999). Robust identification of first-order plus dead-time model from step response, *Control Engineering Practice*, **7**(1), 71-77.