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Multiple Model Networks for Non-linear Modelling and Control

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1 Introduction

Non-linear processes, by their nature, are non-uniform and invariably require custom designed control schemes to deal with individual characteristics. No general theory deals comprehensively with the wide range of non-linear systems encountered. In an attempt to accurately model non-linear dynamical systems, a wide variety of techniques have been developed such as non-linear auto-regressive moving average with exogeneous inputs (NARMAX) models (Chen and Billings, 1989), Weiner models (Schetzen, 1981), Hammerstein models (Billings and Fakhouri, 1982) and Multiple Layer Perceptron (MLP) neural networks (Narendra and Kannan, 1990). While the accuracy of such models offers a potentially significant improvement over linear models, the process control engineer is faced with the difficulty in their more-or-less so-called black-box representation of dynamics of non-linear systems. This back-box representation fails to exploit the significant theoretical results available in the conventional modelling and control domain, making it difficult to analyse the behaviour of the controlled system and to prove its stability.

The last decade has shown an increase in the use of local model representations of non-linear dynamic systems. The basic structure includes a number of approaches: Tagaki-Sugeno (1985) fuzzy systems, local model networks (Johansen and Foss 1993), gain-scheduled control (Shamma and Athans, 1990), the smooth threshold autoregressive (STAR) models of Tong (1990) and the state dependent models of Priestley (1988). The model parameters are obtained from prior knowledge, linearization of a physical model or identified from measured data. The advantages of these approaches are purported to be their simplicity and the insight into global dynamics obtained from their local models. The construction of interpolating the behaviour of locally valid models offers an attractive and intuitively pleasing method of modelling non-linear systems. Moreover, in terms of control, it potentially provides a convenient framework for obtaining both stability and improved performance simultaneously.

Despite the wide applications of multiple model networks in non-linear systems and the growing interest in this area, there is a notable lack of a formal review of the literature. This paper intends to provide a systematic presentation of features, advantages and problems encountered in the application of multiple model networks application in modelling for control. The scope of this paper includes the main theoretical results and brief design procedures relating to multiple model networks with the aim of providing both a critical overview and a useful entry point into the relevant literature. Furthermore, it explores the links between the fields of control science and multiple model networks in a unified presentation and identifies the key areas for future research.

2 Multiple Model Networks

If multiple models are used for non-linear process modelling, techniques for multiple model development need to be developed and in turn, the following issues should be of concern:

- The structure and number of the local models
- The division of local regimes
- Interpolation skills among the local models

The local model structure can employ either linear or non-linear local models. Indeed, for that matter, a mixture of linear and non-linear models maybe used. However, this heterogeneous LM network (Murray-Smith, 1994) would require different training or optimisation techniques for the different classification of local models used. Thus, in general, the same class of local model is used throughout the LM structure. From a practical point of view, the most common choice for the local models is linear. While the linear models are favoured for their representational ability, their simplicity, ease of interpretability and robustness to noisy data has made them very useful in practice as well.

2.1 **Conventional Local Model Networks**

The local model network approach is one promising class of multiple model approaches with interpolation, wherein a small number of relatively simple dynamic systems are, in some sense, blended together. It employs the divide-and-conquer strategy of dividing a complex system into several simpler sub-problems, whose individual solution combine to give the solution to the original problem by interpolation, associated with a corresponding set of weighting functions that defines the validity of the local models. Typically, each simple system is a local linear model or affine model, which describes the dynamics of the non-linear system in some small region of the operating space. The role of blending is to provide smooth interpolation, in some sense, between the local models, with the aim of achieving an accurate representation with only a small number of local models. The blending, therefore, is central to the utility of the approach.

2.1.1 Basic Notion of Local Model Networks

Consider the general non-linear state space system, with state vector $\mathbf{x}$ and input $\mathbf{u}$:

$$\begin{aligned}\dot{\mathbf{x}}(\mathbf{t}) &= \mathbf{f}(\mathbf{x}(\mathbf{t}),\mathbf{u}(\mathbf{t}))\\ \mathbf{y}(\mathbf{t}) &= \mathbf{g}(\mathbf{x}(\mathbf{t}),\mathbf{u}(\mathbf{t}))\end{aligned} \qquad (2.1)$$

where state $\mathbf{x} \in \Re^N$, input $\mathbf{u} \in \Re^p$. For convenience, it is assumed that $\mathbf{y} = \mathbf{Cx}$ without loss of generality, because the output $\mathbf{y}$ is effectively a constant vector multiplied by the state vector. In many cases, the behaviour of a non-linear system near an operating point $(\mathbf{x}_0,\mathbf{u}_0)$ can be described by a linear time-invariant system. To see this, we consider state and input trajectories that are small perturbations away from the operating point:

$$\begin{aligned}\mathbf{x}(\mathbf{t}) &= \mathbf{x}_0 + \boldsymbol{\delta}\mathbf{x}(\mathbf{t})\\ \mathbf{u}(\mathbf{t}) &= \mathbf{u}_0 + \boldsymbol{\delta}\mathbf{u}(\mathbf{t})\end{aligned} \qquad (2.2)$$

where $\mathbf{u}_i$ is the nominal input and $\boldsymbol{\delta}\mathbf{u}(t)$ is the perturbation input. The input and state vector obey the differential equation, determined by submitting (2.2) into (2.1):

$$\boldsymbol{\delta}\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{f}(\mathbf{x}_0 + \boldsymbol{\delta}\mathbf{x}(\mathbf{t}),\mathbf{u}_0 + \boldsymbol{\delta}\mathbf{u}(\mathbf{t})) \qquad (2.3)$$

Expanding the right-hand side of (2.3) in a Taylor series about ($\mathbf{x}_0$, $\mathbf{u}_0$) and keeping only the linear terms yields

$$\boldsymbol{\delta}\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{f}(\mathbf{x}_0,\mathbf{u}_0) + \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\big|_{(\mathbf{x}_0,\mathbf{u}_0)}\, \partial\mathbf{x}(\mathbf{t}) + \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\big|_{(\mathbf{x}_0,\mathbf{u}_0)}\, \partial\mathbf{u}(\mathbf{t}) \qquad (2.4)$$

Defining $\mathbf{A}_0 = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}\big|_{(\mathbf{x}_0,\mathbf{u}_0)}$, $\mathbf{B}_0 = \frac{\partial \mathbf{f}}{\partial \mathbf{u}}\big|_{(\mathbf{x}_0,\mathbf{u}_0)}$, submitting (2.1) to (2.4), we have

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{A}_0\mathbf{x}(\mathbf{t}) + \mathbf{B}_0\mathbf{u}(\mathbf{t}) + \boldsymbol{\alpha}_0 \qquad (2.5)$$

in which, $\boldsymbol{\alpha}_0 = \mathbf{f}(\mathbf{x}_0,\mathbf{u}_0) - (\mathbf{A}_0\mathbf{x}_0 + \mathbf{B}_0\mathbf{u}_0)$. This is an exact model of (2.1) at the point $(\mathbf{x}_0,\mathbf{u}_0)$. At equilibrium point, the constant trend $\boldsymbol{\alpha}_0$ vanishes and the dynamics could be fully captured by $(\mathbf{A}_0,\mathbf{B}_0)$ parameters, which leads to the normal linear local model. The structure of (2.5) contains excessive degrees of freedom $\boldsymbol{\alpha}_0$, which leads to a reasonable approximation in a small neighbour-hood of this point, especially if it is far from equilibrium. It is called an affine local model.

By a blended local model structure we describe a dynamic model of the form

$$\dot{\mathbf{x}} = \sum_i^{Nm} \rho_i(\mathbf{x},\mathbf{u},\mathbf{w})\mathbf{F}_\mathbf{i}(\mathbf{x},\mathbf{u}) \qquad (2.6)$$

where state $\mathbf{x} \in \Re^N$, input $\mathbf{u} \in \Re^p$, the model $\mathbf{F}_\mathbf{i}(:,:)$ is one of *Nm* vector functions of the state and the input, and is valid in a region defined by the scalar validity function $\boldsymbol{\rho}_\mathbf{i}$, which in turn is a function of the above variables. Typically, the local models are chosen to be of the form $\mathbf{F}_\mathbf{i}(\mathbf{x},\mathbf{u}) = \mathbf{A}_\mathbf{i}\mathbf{x} + \mathbf{B}_\mathbf{i}\mathbf{u} + \boldsymbol{\alpha}_\mathbf{i}$, resulting in constituent dynamics systems $\sum_i$ given by $\sum_\mathbf{i}: \dot{\mathbf{x}} = \mathbf{F}_\mathbf{i}(\mathbf{x},\mathbf{u}) = \mathbf{A}_\mathbf{i}\mathbf{x} + \mathbf{B}_\mathbf{i}\mathbf{u} + \boldsymbol{\alpha}_\mathbf{i}$, where $\mathbf{x},\mathbf{d}_\mathbf{i} \in \Re^\mathbf{N}$, $\mathbf{A}_\mathbf{i} \in \Re^{\mathbf{N}\times\mathbf{N}}$, and $\mathbf{B}_\mathbf{i} \in \Re^{\mathbf{N}\times\mathbf{P}}$. This results in a non-linear description of plant dynamics of the form

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x},\mathbf{u},\mathbf{w}) + \mathbf{B}(\mathbf{x},\mathbf{u},\mathbf{w})\mathbf{u} + \boldsymbol{\alpha}(\mathbf{x},\mathbf{u},\mathbf{w})$$

where

$$\mathbf{A}(\mathbf{x},\mathbf{u},\mathbf{w}) = \sum_{i}^{Nm} \boldsymbol{\rho}_\mathbf{i}(\mathbf{x},\mathbf{u},\mathbf{w})\mathbf{A}_i$$

$$\mathbf{B}(\mathbf{x},\mathbf{u},\mathbf{w}) = \sum_{i}^{Nm} \boldsymbol{\rho}_\mathbf{i}(\mathbf{x},\mathbf{u},\mathbf{w})\mathbf{B}_\mathbf{i}$$

$$\boldsymbol{\alpha}(\mathbf{x},\mathbf{u},\mathbf{w}) = \sum_{i}^{Nm} \boldsymbol{\rho}_\mathbf{i}(\mathbf{x},\mathbf{u},\mathbf{w})\boldsymbol{\alpha}_\mathbf{i}$$

Here, $\boldsymbol{\alpha}_\mathbf{i}$, $\mathbf{A}_\mathbf{i}$ and $\mathbf{B}_\mathbf{i}$ are all constants. Model building consists of covering the operating space of the non-linear plant with local models. Behaviour along the plant equilibria is typically captured by using models whose equilibria are located inside the region defined by their basis functions, whereas behaviour off equilibria can be properly approximated by using affine basis functions. It is shown in (Johansen et al. 1998) that the finite set of linearizations about a finite number of points (equilibria and transient points) can be used to accurately approximate dynamic linearization about arbitrary trajectories, using an interpolated multiple model structure with local affine dynamic models.

2.1.2 Linear Local Modelling

Conventionally, dynamical modelling of non-linear systems has been carried out on the basis of linearisation about equilibria (ARMAX models) and much of control theory is based on the use of linear models. Local linear model (LLM) networks fit neatly into this area of work, as they can be regarded as forming local ARMAX models, which are interpolated to a give a global non-linear model (Johansen and Foss, 1993). This transparency of representation facilities the ease of incorporating a priori knowledge, such as known linear regimes and linear models, into the network process (Gawthrop, 1995). Furthermore, it is compatible with conventional modelling and control skills, for example, conventional internal model control (Brown et al., 1997), predictive control (Irwin and Townsend, 1999) and model-based control (Irwin 1998). In particular, it is claimed to be relatively straightforward to design a local controller network once the LLM network has been defined (Townsend et al. 1998, Townsend and Irwin, 1999).

LLM networks clearly inherit many valuable properties. However, a LM network using strictly local linear models, in conjunction with normalised basis functions, can result in a poor global representation of the non-linear plant being approximated. Since normalisation causes the gaussian functions to sum to unity over the entire operating space, the steady-state output of the LM network is restricted to lie within that part of space bounded by the local models (McLoone, 2000). It can be easily seen, despite any improvement in the normalised weighting functions or change in the positioning of the local linear models, the LM network will not accurately represent the non-linear plant in the region beyond the operating points. Introducing more linear local models into the network will reduce the actual steady-state error, but there will always exist some residual error that cannot be eliminated using the current local modelling approach.

2.1.3 Affine Local Modelling

With the problem on interpretability of LLM as discussed before, the modelling accuracy of LLM networks for non-linear dynamics is questionable. Dynamic linearization, which means the linearization is done based on a nominal trajectory, was suggested in (Driankov et al., 1996), however, a drawback is that the control design for the resulting linear time-varying (LTV) system is in general a very difficult problem. An alternative is off-equilibrium linearization (Johansen et al., 1998), which can be seen depend only on the granularity of the set of points in the off-equilibrium linearization. The reason for this is that the LTV system resulting from dynamic linearization depends only on the point the trajectory passes through at a given time. Hence, off-equilibrium linearization leads to an arbitrarily close approximation of the LTV system in terms of a set of linear time invariant (LTI) systems, provided there exists an LTI system close to any point in the nominal trajectory of the LTV system, and the LTI system are interpolated using a sensible interpolation scheme (Driankov et al., 1996). Mathematically, off-equilibrium linearization leads to local affine models, which have an extra degree of freedom, i.e., an added bias term to make the local models more flexible, so that they can be shifted upwards or downwards in the operating space (McLoone, 2000). This scheme improves the modelling accuracy of LLM significantly.

However, the affine models do not possess the superposition property fundamental to linear systems, due to the inhomogeneous term $\boldsymbol{\alpha}_\mathbf{i}$. Thus, there is a lack of continuity with established linear theory and methods. Furthermore, the inhomogeneous term can become quite large and significantly influence the solution. Then $\boldsymbol{\alpha}_\mathbf{i}$ tends to dominate, while varying some elements, like $\mathbf{A_i}$ and $\mathbf{B_i}$ only have minor influence on the local model accuracy (Shorten et al., 1999). Therefore, in general, the inhomogeneous term cannot be simply regarded as a small approximation error or disturbance, as has been the practice in early-published research (Brown et al., 1997). A novel velocity-based linearization approach is suggested as an alternative (Leith and Leithead, 1999).

2.2 Interpolation Schemes

The use of a set of local linear models has been suggested by several authors, including the suggestion on the weighting function selection and the optimisation of the interpolation schemes. Johansen and Foss (1995) and Nelles (1997) propose construction algorithms from both computational and performance points of view. Brown et al. (1997) introduce hybrid-learning schemes in model optimisation, which act towards minimizing the global error in the space of local models. McGinnity and Irwin (1999) compare the hybrid optimisation algorithm of McLoone et al. (1998 a, b) and the construction algorithm of Johansen and Foss (1995).
The role of blending is to provide smooth interpolation, in some sense, between the local models with the aim of achieving an accurate representation with only a small number of local models. The schemes of interpolation are of significance to the utility of the approach.

Ideally, it is desirable that a multiple model network should give accurate global non-linear prediction and at the same time that its local models are close approximations to the local linearization of the non-linear dynamic system. The former is of significance in the global performance of the modelling; while the latter is as well important in many applications where the constituent local models are used individually and aids validation and interpretation of the model considerably. This requirement defines a multi-objective identification problem, namely, the construction of a dynamic model that is a good approximation of both the local and global dynamics of the underlying system. While these objections are normally conflicting. Moreover, in practice, it is important, for some application, that the global behaviour of the non-linear model is similar to the global behaviour of the non-linear system. For example, this is typically the case when the global model is used for non-linear predictions (Townsend et al., 1998, Townsend and Irwin, 1999) or when the global model is used as an internal model in a controller as in e.g. (Wang, 1993, Brown et al., 1997). On the other hand, it is sometimes required (and often desirable) that the local linear models are accurate approximations to the local linearization. This is the case when the multiple models are used to designing linear local controllers (Narendra and Balakrishan, 1994, Narendra et al., 1995).

In brief, the optimisation of interpolation schemes is closely related to the objective of controller design. The role of interpolation is to balance the trade-off between local and global approximation accuracy based on the methodology adopted for controller design. So far, there are no general methods available to assist in the selection of an appropriate validity function, and related to this, appropriate interpolation schemes towards controller realisation, although some initial work by some researchers has been done (Brown et al., 1997).

2.3 Local Controller Networks

Once the LM network has been formulated, local controller (LC) networks, the control version of LM networks are defined in turn. In general, the global control signal is determined by

$$u(t) = \sum_{i=1}^{n_\phi} \mathbf{C_i}\left(\psi^c(t)\rho_i\left(\tilde{\phi}(t)\right)\right)$$

where $\mathbf{C_i}$ denotes the local controller for each local model $\mathbf{F_i}$. The n_ϕ local controllers thus obtained are blended using the same validity function $\boldsymbol{\rho}_\mathbf{i}$, which is used in the LM network. The controller information vector $\boldsymbol{\psi}^c$ consists of past control inputs, current and past plant outputs, and the current and past values of the reference signal $\mathbf{y_{ref}}$. The basic idea of the LC networks is to adaptively blend various controllers at different operating regions of the process in a proper way through a gating system. The gating system $\boldsymbol{\rho}_\mathbf{i}$ results from the approach formulating the LM network.

From a control engineering perspective, the use of LC networks bridge the gap between multiple model control and conventional control. Many existing tools and theories in linear systems can be organised in local controller network structures for dynamic non-linear system control. However, there is a pressing requirement for the proof of stability and robustness from the global performance point of view. Another important issue is on the design and optimisation of interpolation systems, which is in fact, in some ways, an issue of modelling as we discussed in previous sections.

2.4 Piecewise Linear Models for Control

The aim of this section is to show that the selection of local models and local controllers does not necessarily imply a blending or interpolation. The importance of gating the input vectors according to the performance of the modelling or control error has been highlighted in previous sections. It is clear, if the current performance of each model regarding a certain input vector is available, that the selection of the valid model instantly would be straightforward by having access to the modelling error acting on system as a criterion.

The use of local linear models without interpolation, i.e. piecewise linear models, have been suggested by several authors, including Skeppstedt et al. (1992), Billings and Voon (1987), and Tong and Lim (1980). A related technique is the use of splines (Friedman 1991) for representing dynamic models (Psichogios et al., 1992). splines are also local models, but unlike piecewise linear models, there are constraints that enforce smoothness on the boundaries between the local models.

A controller is often highly dependent on a plant model especially when the controller has been designed out of the model. Hence, for those cases, the modelling error would be a relevant criteria for controller selection. If the number of controllers is bounded, the delay between the selection of the controller and its activation can be negleted. Thus the selection of the controller according to the modelling error is feasible. This idea has been used by (Narendra et al, 1995; Narendra and Balakrishan, 1994) to develop the Multiple Switched Model (MSM) scheme. However, such a multiple controller scheme was previously introduced by (Middleton et al., 1988) and further extended in (Morse, 1990; Morse et al., 1992, Weller and Goodwin, 1994) and labelled the "Hysteresis switching algorithm". This algorithm aims at achieving stability whereas the MSM is used for improving the control performance whilst dealing with a process having its parameters changing quickly with time.

In brief, the MSM has proved to be capable of handling plants with rapid change of parameter values or with highly non-linear characteristics. This method is not sensitive to the input space dimension as the gating system is clustering free. The stability and robustness proofs (Narendra and Balakrishnan 1997, Narendra and Xiang 2000) give to this scheme a necessary credibility. Finally, the constructive algorithm used to determine the required number of model-controller pairs gives a complete autonomy to this control scheme. The MSM is clearly one of the most powerful non-linear controllers ever developed where the performance of a controller is related to the performance of a model (i.e. in cases where controllers have been designed out of models).

3 Velocity-based Local Model Networks

A static model gives information about the steady state relation between the input and the output signal. A dynamic model should give the relationship between the input and the output signal during transients. It is naturally much more difficult to capture dynamic behaviour. In an attempt to accurately model nonlinear dynamical systems, a wide variety of techniques have been developed such as nonlinear auto-regressive moving average with exogenous inputs (NARMAX) models (Chen and Billings, 1989), Weiner models (Schetzen, 1981), Hammerstein models (Billings and Fakhouri, 1982) and Multiple Layer Perceptron (MLP) neural networks (Narendra and Kannan, 1990). However, all of these methods have difficulty in exploiting the significant theoretical results available in conventional modelling because of their so-called black-box representation of nonlinear systems.

In contrast, Local Model (LM) networks were proposed as a modelling frame that could produce highly transparent models (Johansen, 1993). It was purported that the locally valid sub-models were easily interpreted and that the weighted sum of the local sub-models provided a qualitative high-level description of the nonlinear system.

However, recent research has questioned the ease of interpretability of the multiple model frameworks, demonstrating that the global dynamics of the conventional LM network are only weakly related to the dynamics of the underlying local models. Leith and Leithead (1999) presented a novel class of blended multiple-model networks whereby the global dynamics are directly related to the local models employed. Moreover, the underlying sub-models are continuous-time, velocity-based and linear, thus ensuring continuity with existing linear techniques, which is useful for analysis and controller design. Furthermore, analytical results based on the complex non-linear continuous stirred tank reactor (CSTR) process show that the velocity-based approach is ideally suited to the development of local controller (LC) networks (McLoone, 2001).

So far, a lot work has been done regarding the conventional LM technique in both the continuous-time and discrete-time domains. However, all the studies relating to velocity-based LM networks exist in the continuous-time domain. Considering the popular applications of digital computer in the field of control and the potential capability of velocity-based LM network approach in the development of LC networks, a discrete-time version of the velocity-based multiple model representation has been developed (Gao et al., 2002). The modelling capabilities of the resulting non-linear model are examined using a highly complex non-linear process, in the form of a simulated continuous stirred tank reactor.

4 Conclusion

It is worth mentioning that it is not normally obvious how to design a controller from a non-linear model even if this model gives a transparent representation of the plant. Hence, for the purpose of control, the model does not only have to yield an interpretable representation of the plant, it must be oriented to the controller design. The controller theory is, at this stage, only well-developed and understood for linear systems. Since it is well known how to design a controller from a linear model, the multiple linear modelling approaches, used in both the LM network and the multiple switched models, can be considered to be one of the few non-linear modelling approaches oriented to controller design.

Other significant advantages arise from the use of LM network in general. The LM network tends to yield interpretable representation whereas MLP-like approaches are black box in nature and converge very slowly towards a solution or simply fail to learn. It is possible to incorporate a priori knowledge into LM networks. Another very important feature is that LM network has been proved to be capable of approximating any kind of functions. These significant advantages reveal the powerful potential of the LM network approach for non-linear modelling and control. Moreover, the velocity-based LMN has better capability in capturing the dynamics than the conventional LM networks. This brings promising potential for its application in controller design. Further work is required on local controller networks design based on the developed discrete velocity-based LM network.

5 Timetable outline

So far, Most of works in the schedule have been done. They have been organized in papers published ([41],[42],[43]). Some improvement on the presented approaches may be needed to work parallel with the thesis write-up. I suppose to finish the thesis draft at the end of the year 2002, and hope to submit the final thesis in February 2003.

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