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Padraig Dowds
Dublin Institute of Technology

Aidan O'Dwyer
Dublin Institute of Technology, aidan.odwyer@dit.ie

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Modelling and Control of a Suspension System for Vehicle Applications

P. Dowds and A. O'Dwyer *
Faculty of Engineering, Dublin Institute of Technology, Kevin St., Dublin 8.
* Principal author: e-mail - aidan.odwyer@dit.ie

Abstract: This paper discusses the modelling of ***passive*** and ***active*** suspension systems in a car, and the subsequent design of appropriate feedback controllers for the active suspension system. The models will be investigated using a quarter car model and a full car model approach.

1. Introduction

The main objective of suspension systems is to reduce motions of the sprung mass (vehicle body) to road disturbances. Conventional vehicle suspension systems achieve this through passive means using springs and dampers. When designing vehicle suspensions, the dual objective is to minimise the vertical forces transmitted to the passenger, and to maximise the tyre-to-road contact for handling and safety. While traditional passive suspension systems can negotiate this trade-off effectively, active suspension systems have the potential to improve both ride quality and handling performance, with the important benefits of better braking and cornering because of reduced weight transfer. This improvement is conditional upon the use of feedback control of the actuators in the active suspension system.

2. Modelling – quarter car

The quarter car model is set up using interconnections of masses, springs and dampers. Figures 1 and 2 show a passive and active quarter car suspension system model, respectively. In these diagrams, M_s is the mass of the car body, M_{us} is the unsprung mass of the wheel and axle assembly, k_s is the spring constant in the suspension system, b_s is the dashpot constant (representing the shock absorber) and k_t is the spring constant of the tyre (the values of these parameters are taken from [1]); r represents the road input force, x_s represents the force acting on the mass M_s, x_{us} represents the force acting on the mass M_{us} and f represents the control element (Figure 2 only).

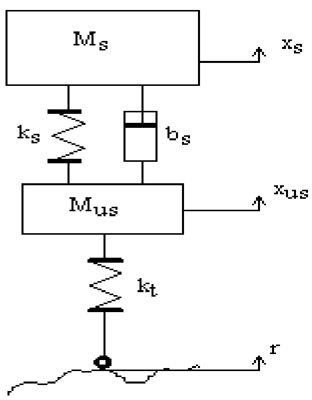

Figure 1: Quarter car model – passive

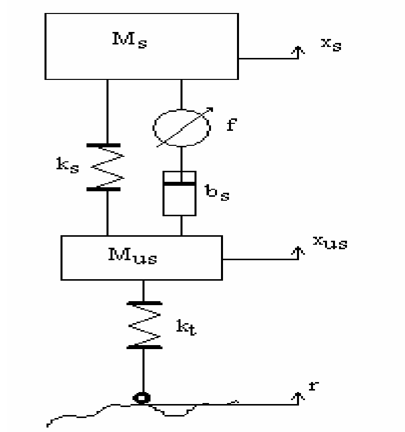

Figure 2: Quarter car model – active

For the passive quarter car model, the state equations may be deduced to be:

$$M_s \ddot{x}_s = -k_s(x_s - x_{us}) - b_s(\dot{x}_s - \dot{x}_{us}) \quad (1)$$

$$M_{us} \ddot{x}_{us} = k_s(x_s - x_{us}) + b_s(\dot{x}_s - \dot{x}_{us}) - k_t(x_{us} - r) \quad (2)$$

These equations give the following transfer function relating x_s to r:

$$\frac{x_s}{r} = \frac{k_t(b_s s + k_s)}{M_s M_{us} s^4 + (M_s + M_{us}) b_s s^3 + ((M_s + M_{us}) k_s + M_s k_t) s^2 + b_s k_t s + k_s k_t} \quad (3)$$

For the active quarter car model, the state equations are:

$$M_s \ddot{x}_s = -k_s(x_s - x_{us}) - b_s(\dot{x}_s - f) \quad (4)$$

$$M_{us} \ddot{x}_{us} = k_s(x_s - x_{us}) + b_s(f - \dot{x}_{us}) - k_t(x_{us} - r) \quad (5)$$

These equations give the following transfer function relating x_s to r and f:

$$x_s = \frac{k_s k_t}{M_s M_{us} s^4 + (M_s + M_{us}) b_s s^3 + ((M_s + M_{us}) k_s + b_s^2 + M_s k_t) s^2 + (2k_s + k_t) b_s s + k_s k_t}.r$$

$$+ \frac{M_{us} b_s s^2 + b_s^2 s + (2k_s + k_t) b_s}{M_s M_{us} s^4 + (M_s + M_{us}) b_s s^3 + ((M_s + M_{us}) k_s + b_s^2 + M_s k_t) s^2 + (2k_s + k_t) b_s s + k_s k_t}.f \qquad (6)$$

3. Modelling – full car

Figures 3 and 4 show two passive full car suspension system models, with Figure 4 including the effects of the pitch and roll motions.

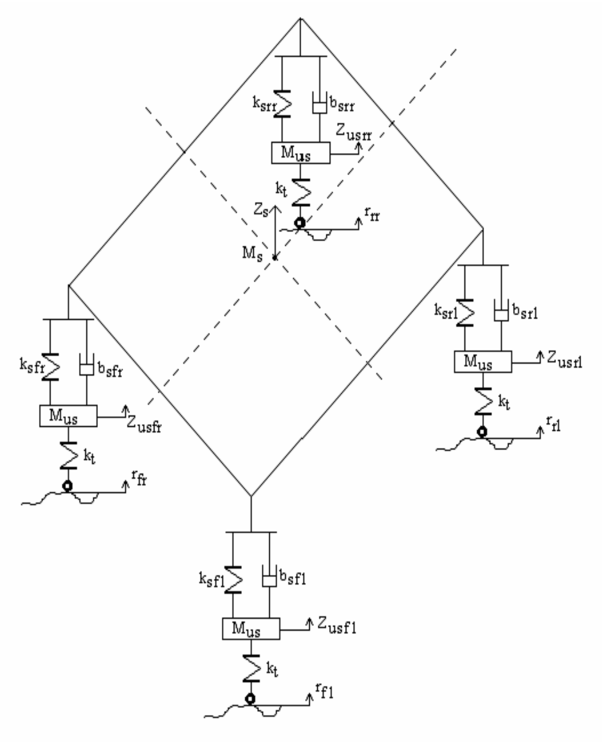

Figure 3

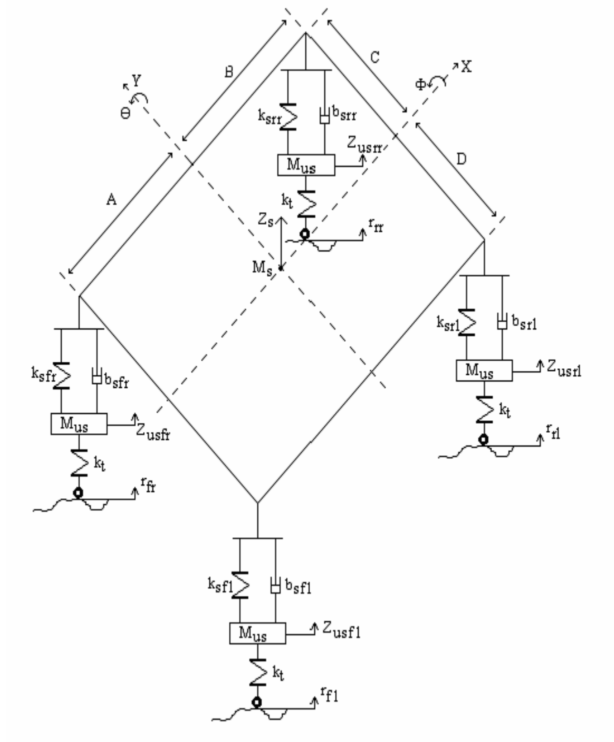

Figure 4

In these diagrams, the coefficient labels mirror those of Figures 1 and 2; for Figure 4, θ represents the pitch angle of the car and ϕ represents the roll angle of the car. The values of the parameters are taken from [2]. For Figure 3, the state equations may be deduced to be

$$\dot{x}_2 = \frac{1}{M_s}[-(2k_{sf} + 2k_{sr})x_1 - (2b_{sf} + 2b_{sr})x_2 + k_{sf}x_3 + b_{sf}x_4$$
$$+k_{sf}x_5 + b_{sf}x_6 + k_{sr}x_7 + b_{sr}x_8 + k_{sr}x_9 + b_{sr}x_{10}]$$

$$\dot{x}_4 = \frac{1}{M_{us}}[k_{sf}x_1 + b_{sf}x_2 - (k_{sf} + k_t)x_3 - b_{sf}x_4 + k_t r_{fl}]$$

$$\dot{x}_6 = \frac{1}{M_{us}}[k_{sf}x_1 + b_{sf}x_2 - (k_{sf} + k_t)x_5 - b_{sf}x_6 + k_t r_{fr}]$$

$$\dot{x}_8 = \frac{1}{M_{us}}[k_{sr}x_1 + b_{sr}x_2 - (k_{sr} + k_t)x_7 - b_{sr}x_8 + k_t r_{rl}]$$

$$\dot{x}_{10} = \frac{1}{M_{us}}[k_{sr}x_1 + b_{sr}x_2 - (k_{sr} + k_t)x_9 - b_{sr}x_{10} + k_t r_{rr}], \qquad (7)$$

with $\dot{x}_1 = x_2$, $\dot{x}_3 = x_4$, $\dot{x}_5 = x_6$, $\dot{x}_7 = x_8$ and $\dot{x}_9 = x_{10}$. To develop these equations, $k_{sfl} = k_{sfr} = k_{sf}$, $k_{srl} = k_{srr} = k_{sr}$, $b_{sfl} = b_{sfr} = b_{sf}$ and $b_{srl} = b_{srr} = b_{sr}$. The state variables are assigned as follows: $x_1 = z_s$, $x_2 = \dot{z}_s$, $x_3 = z_{usfl}$, $x_4 = \dot{z}_{usfl}$, $x_5 = z_{usfr}$, $x_6 = \dot{z}_{usfr}$, $x_7 = z_{usrl}$, $x_8 = \dot{z}_{usrl}$, $x_9 = z_{usrr}$ and

$x_{10} = \dot{z}_{usrr}$. More involved state space equations may be deduced for Figure 4. Similarly, figures 5 and 6 show two active full car suspension system models, with Figure 6 including the effects of the pitch and roll motions. The coefficient labels mirror those of Figures 1 to 4.

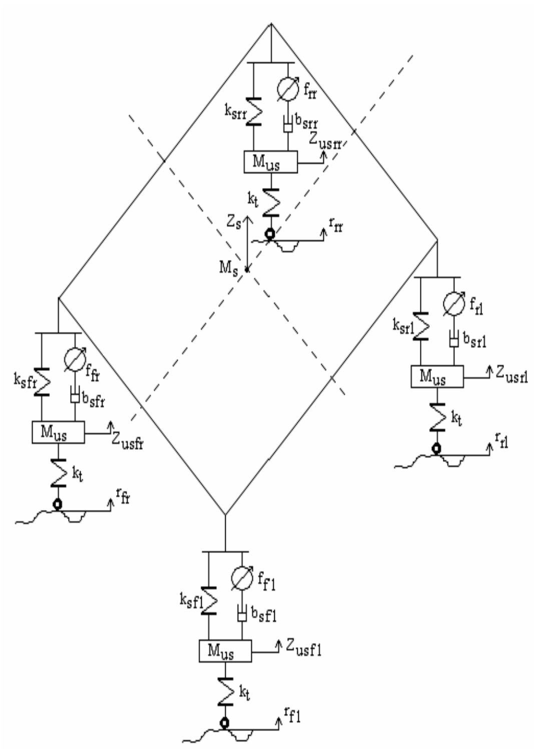

Figure 5

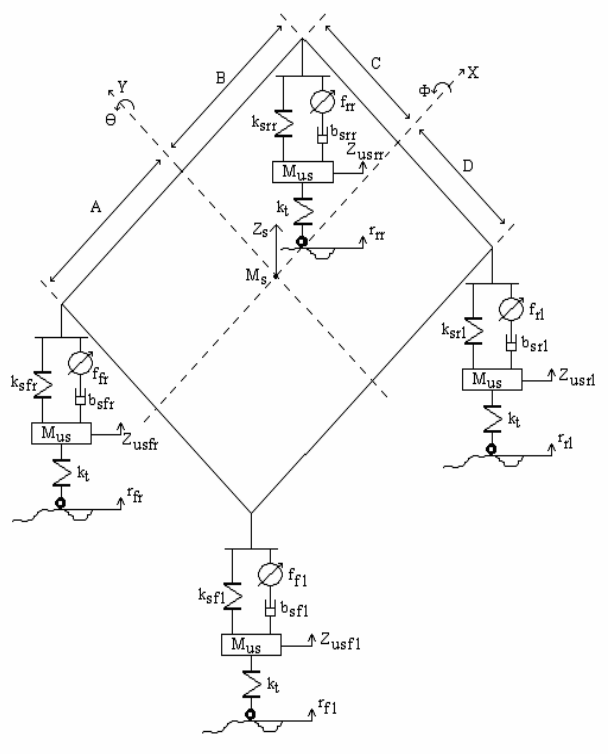

Figure 6

For Figure 5, the state equations may be deduced to be

$$\dot{x}_2 = \frac{1}{M_s}[-(2k_{sf} + 2k_{sr})x_1 - (2b_{sf} + 2b_{sr})x_2 + k_{sf}x_3 + k_{sf}x_5$$

$$k_{sr}x_7 + k_{sr}x_9 + b_{sf}f_{fl} + b_{sf}f_{fr} + b_{sr}f_{rl} + b_{sr}f_{rr}]$$

$$\dot{x}_4 = \frac{1}{M_{us}}[k_{sf}x_1 + b_{sf}f_{fl} - (k_{sf} + k_t)x_3 - b_{sf}x_4 + k_t r_{fl}]$$

$$\dot{x}_6 = \frac{1}{M_{us}}[k_{sf}x_1 + b_{sf}f_{fr} - (k_{sf} + k_t)x_5 - b_{sf}x_6 + k_t r_{fr}]$$

$$\dot{x}_8 = \frac{1}{M_{us}}[k_{sr}x_1 + b_{sr}f_{rl} - (k_{sr} + k_t)x_7 - b_{sr}x_8 + k_t r_{rl}]$$

$$\dot{x}_{10} = \frac{1}{M_{us}}[k_{sr}x_1 + b_{sr}r_{rr} - (k_{sr} + k_t)x_9 - b_{sr}x_{10} + k_t r_{rr}] \quad (8)$$

with $\dot{x}_1 = x_2$, $\dot{x}_3 = x_4$, $\dot{x}_5 = x_6$, $\dot{x}_7 = x_8$ and $\dot{x}_9 = x_{10}$. The state equations are developed in a corresponding manner to that of Figure 3.

4. Controller design – active suspension system

An indicative result of simulation work that evaluates the performance of the active suspension system when the feedback controllers are designed using a number of techniques, using the performance of the passive suspension system as a benchmark, is now reported; the simulation work is carried out using MATLAB/SIMULINK. A road disturbance (e.g. a pothole) is simulated; recovery from the disturbance is plotted for the passive suspension system, and the active suspension system when the controller is implemented as a proportional (P) controller or a proportional-integral (PI) controller. The controllers are designed using a standard root locus technique to achieve a +/-2% settling time of one second. Figure 7 shows the simulated disturbance responses.

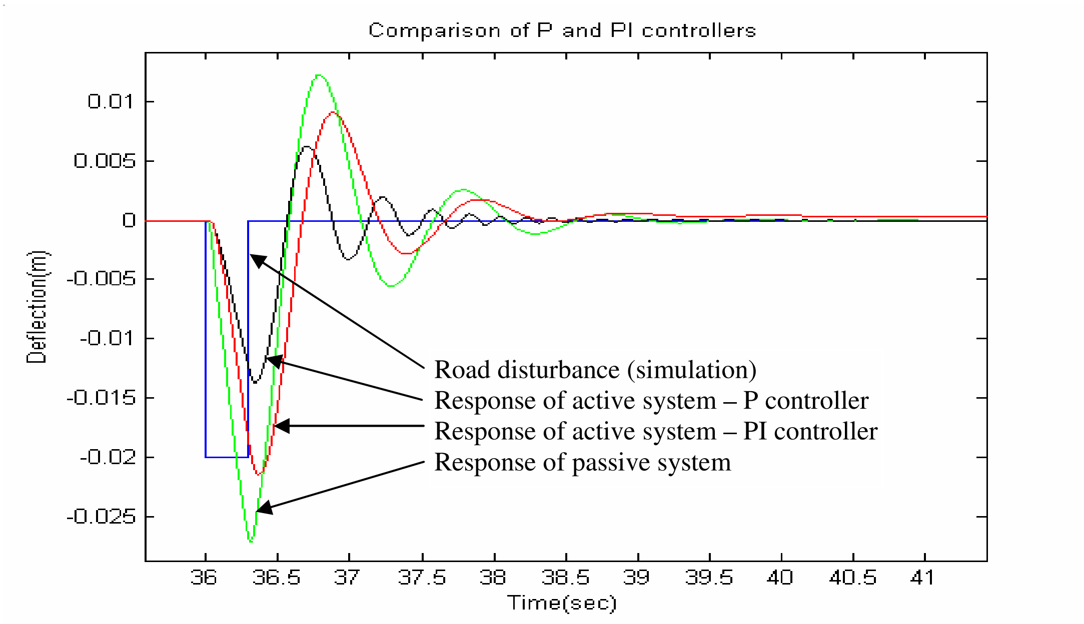

Figure 7

The results show that the active suspension system allows significant improvement over the passive system. Further such results will be discussed at the symposium.

5. Conclusions

The paper reports on the modelling of passive and active suspension systems, for both a quarter car and full car model. It is shown that the active suspension system facilitates significantly improved regulator response when compared to the passive suspension system. The controlling element of the active suspension system is generally based on an actuator; the main practical difficulty in implementing active suspension is the power consumption of the actuator.

References

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[2] Lakehal-Ayat, M., Diop, S. and Fenaux, E. (2002). "Development of a full active suspension system". Paper 2658, *Proceedings of the IFAC World Congress*, Amsterdam.