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A Suggested Building Block Geometry Without Continuous Radial Joints of Possible Relevance for Particle Detectors

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Configuration of Rhombohedral Elements

First consider a cubic configuration using two colours to highlight the cubes and to show that the cubes fit together in 3-D like the black-and-white squares on a chess board in 2-D. See Fig. 1

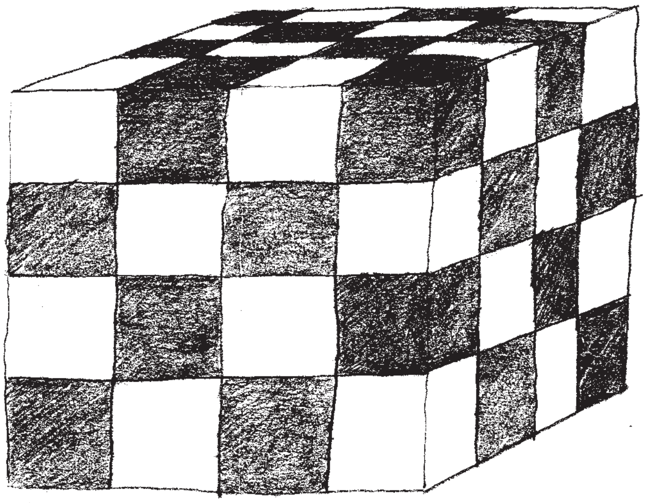
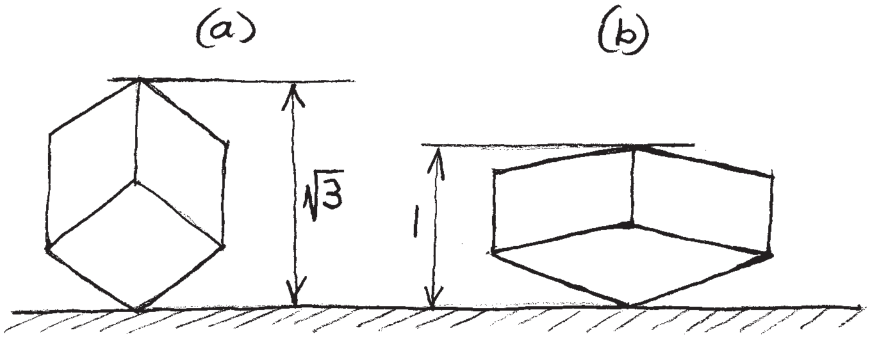


Fig. 1
Cubic lattice with black and white unit elements

Assume each small cube has unit edge length. Each small cube has four body diagonals of length $\sqrt{1^2+1^2+1^2} = \sqrt{3}$. Every point on the cubic lattice is a vertex for eight small cubes (four black and four white).

The cubic lattice is highly important because of its symmetry, but there is a particular rhombohedral lattice that is highly symmetrical and is also highly important, although it has not attracted a lot of attention so far. The unit cell (a rhombohedron), like the cube, has twelve unit edges. However, unlike the cube, one of its body diagonals has unit length, while the other three body diagonals are of equal length to one another, but longer than those of the cube.

An easy way to visualise the (unit diagonal) rhombohedral unit cell or lattice is to imagine distorting a wire-framed cubic unit cell or cubic lattice. See Fig. 2.



Unit cube with one of its body diagonals vertical

Unit rhombohedral cell with its unit-diagonal vertical

Fig. 2 Rhombohedral unit cell as a distorted cubic cell

A regular rhombohedral lattice can be formed by packing unit rhombohedral cells together such that there are no voids. Black and white rhombohedra can be used and the blocks fit together in the same configuration as illustrated for the cube in Fig. 1. The vertices of the rhombohedra are the points that comprise a regular rhombohedral point lattice. The edges of the packed rhombohedra are possible unit-length links between the nodes of the rhombohedral point lattice.

Thus far it can be appreciated that the rhombohedral point lattice (consisting only of points distributed regularly in space) is a distorted cubic lattice in which one unit-cell body diagonal has been shortened to have unit length, while the remaining three body diagonals have been stretched. It would seem to be less symmetrical than the original cubic point lattice. However, the sacrifice of the symmetry between the four body diagonals enables a new type of symmetry of the rhombohedral point lattice about a point and about a line.

If all of space is filled with a rhombohedral point lattice as has been described, the space within the points could be filled without voids by solid rhombohedral unit-sided elements that all have the same orientation, i.e. the single unit-length body diagonal of every rhombohedron has the same direction. All unit-length body diagonals are parallel. It is a characteristic of this special rhombohedral point lattice that each point in the lattice has eight nearest neighbours (at unit distance). These eight comprise four pairs where the members of the pair are mirror images of each other through the original point of which they are neighbours.

As a consequence of the symmetrical distribution of the eight nearest neighbours of any point on the regular rhombohedral point lattice there are four possible orientations of the individual unit rhombohedra that could be used to fill the entire point lattice with rhombohedral unit cell solids all having the same orientation and without any voids. Each of the four would have the same two-colour configuration as shown in Fig. 1.

The real magic of this rhombohedral lattice is that it is possible to fill all of space with unit-sided rhombohedra using all four possible orientations of the rhombohedra for the same rhombohedral point lattice in such a way that space is partitioned into four symmetrical regions, one for each unit-body-diagonal orientation. Each of the four regions or partitions has the two-colour configuration that is shown in Fig. 1. The four regions touch at just one single point where just four rhombohedra (and the unit-body-diagonals of the four) meet. There are also four lines along which, at lattice points, six rhombohedra meet. There are six flat planes that partition the four regions. At each of these planes a rhombohedron from one of the four regions makes face-contact with a rhombohedron from one of the other regions. The black-face-contacts-white-face rule that is embodied in Figure 1 is not complied with over all six partition faces.

The partitioned rhombohedral lattice with four regions of rhombohedral solid elements has 3-way rotational symmetry about each of the four lines mentioned. A cross-section through the rhombohedral elements around one of these lines is shown in Fig. 3.

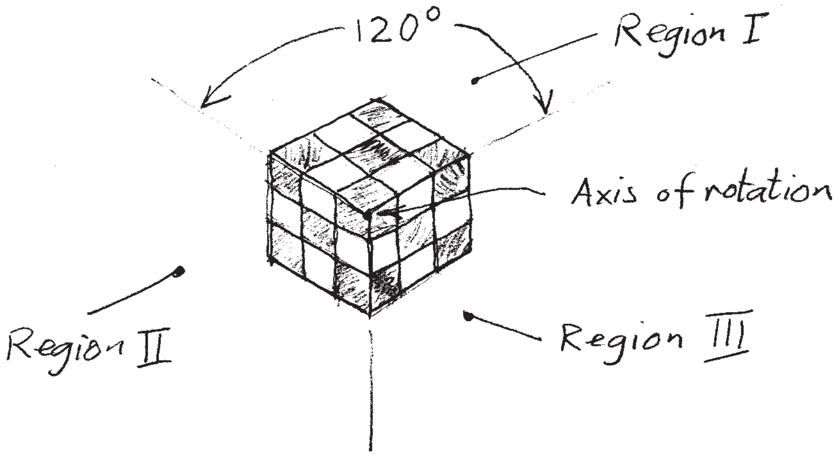


Fig. 3 Cross section through one of the four principal axes of the partitioned rhombohedral lattice.

It can be noted from Fig. 3 that the black-face-contacts-white-face rule that is embodied in Fig. 1 is not complied with, prima faciae, at the six partitioning surfaces of the partitioned rhombohedral lattice. However, if each of the four regions is given its own pair of colours (making 8 colours in all then the rule 'a rhombohedron always makes face contact with a rhombohedron of a different colour' is complied with throughout the partitioned rhombohedral lattice.[1] Whether or not colours are used does not affect the geometry of the configuration.

the partitioned rhombohedral lattice has 4-way rotational symmetry about the unique point where the four partitioned regions meet. The four principal axes of rotation radiate from that point and are symmetrically distributed, as shown in Fig. 4(a). If the four axes are drawn with equal lengths, the four outer ends define a regular tetrahedron as in Fig. 4 (b).

(a) (b)

Fig. 4 Principal axes of the partitioned rhombohedral lattice

when four unit rhombohedra are placed in contact, one within each of the four regions of the partitioned rhombohedral lattice, the external shape produced has $3 \times 4 = 12$ rhombic faces. There are eight vertices where three edges meet and there are six nodes where four edges meet.

Progressively larger rhombic dodecahedra can be made by adding successive layers of rhombohedra.

As already mentioned, the core rhombic dodecahedron consists of four rhombohedra. To start the next layer four more rhombohedra having the same orientations as the original four are added, touching the outer ends of the unit body diagonals of the original four (point contact only). These provide four of the vertices of the second shell rhombic dodecahedron. They also provide part of each of the twelve faces. Three more rhombohedra can be added to support each of the four just added, but these interlock in such a way that they complete the second layer, i.e. the second layer contains $4 + 4 \times 3 = 16$ rhombohedra. Eight of the vertices have a radius of 2 units. (The remaining 6 have larger radii)

Third and subsequent layers can be added by following a similar procedure. The number of rhombohedra in the third layer is $4 + 4 \times 12 = 52$ and the third layer has eight vertices at a radius of 3 units.

(a) (b)

Fig.5 Constructing the second layer of the rhombic dodecahedron. In (a) the shaded rhombohedron has been placed over a unit-body-diagonal vertex of the core rhombic dodecahedron and three further rhombohedra have been added to support it. In (b) the completed view of three faces of the second layer is shown.

Conclusion

Using just one rhombohedral block type it is possible to build nested hexagonal tubes. By reversing the direction of each successive tube all radial leakage paths (in 3-D) can be intercepted. See Fig. 6

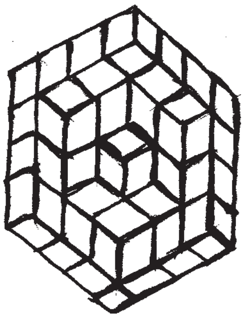


Fig. 6 Construction of nested hexagonal tubes with reversal of successive layers to overlap radial joins.

The construction shown in Fig. 6 is probably the best possible approximation to nested cylinders that can be obtained using one single type of element. Note also that just as eight cubes can be assembled to form a cube, eight rhombohedra can be assembled to form a rhombohedron. This provides additional scaling flexibility.

In a somewhat analogous way, rhombic dodecahedral shells are probably the best possible approximation to nested spherical shells that can be obtained using one single type of element (the rhombohedron with unit edges and one unit body diagonal). These shells can be built-up reversing successive layers to cover all radial joins.

The hexagonal tubes are in fact contained within the built-up rhombic dodecahedral solid — they are centred on the four principal axes.

Endnote

(1) In the rhombohedral point lattice four orientations of a rhombohedral element are possible and there can be 'black or white' (or 'even or odd'). In fact all four orientations are present simultaneously in empty space or 'the vacuum'. If different colours are used to represent evenness or oddness then the vacuum could be regarded as containing eight types of volume elements which have symmetry under certain displace-

ments, rotations and reflections. However in our everyday experience solid objects cannot overlap.

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